STUDIJSKI PROGRAM ENERGETIKA, ELEKTRONIKA I TELEKOMUNIKACIJE

K	TET	Osnovi elektrotehnike 1 (I kolokvijum)	K1
Katedra za teorijsku elektrotehniku www.ktet.ftn.uns.ac.rs			16.09.2017.

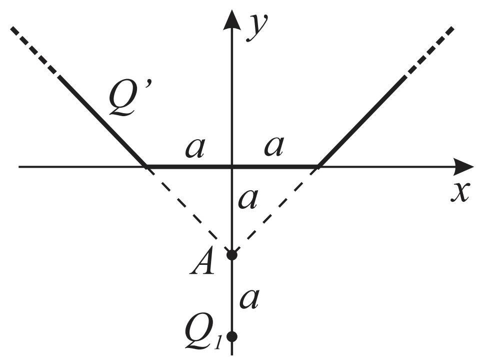
ZADACI

Zadatak 1. Veoma dugačak tanak štap, naelektrisan ravnomerno podužnim naelektrisanjem Q', postavljen je u x-y ravan, Dekartovog koordinatnog sistema, kao što je prikazano na slici 1. Štap se sastoji od dva polubeskonačna segmenta i jednog segmenta konačne dužine, koji leži na x osi.

a) Izvesti, u opštim brojevima, izraz za vektor jačine električnog polja koji u tački A stvara štap.

b) Odrediti količinu tačkastog naelektrisanja, Q_1, koje se nalazi na y osi, tako da rezultantni vektor jačine električnog polja u tački A bude duplo veći od vektora jačine električnog polja koji stvara štap u tački A.

Brojni podaci su: $a = 1\ cm$, $Q' = 10\ nC/m$, $\varepsilon_0 = 8{,}85\cdot10^{-12}\ F/m$.



Slika 1.

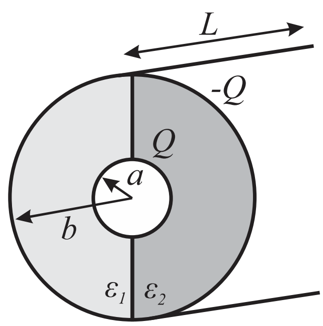
Zadatak 2. Koaksijalni kabl, prikazan na slici 2, poluprečnika elektroda $a = 3\ mm$ i $b = 5\ mm$, dužine $L = 2\ m$, ispunjen je sa dva sloja dielektrika, permitivnosti $\varepsilon_1 = 2\varepsilon_0$ i $\varepsilon_2 = 9\varepsilon_0$. Elektrode kondenzatora su naelektrisane naelektrisanjem Q i $-Q$.

a) Razmotriti granične uslove i odrediti kako se u zavisnosti od rastojanja tačke od centra kondenzatora menjaju intenziteti vektora električnog pomeraja i vektora jačine električnog polja.

b) Izračunati brojnu vrednost kapacitivnosti kabla.

c) Izračunati maksimalni napon na koji sme da se priključi kabl.

Ostali brojni podaci: $E_{č1} = 65\ kV/cm$, $E_{č2} = 95\ kV/cm$.



Slika 2.

PRAVILA POLAGANJA

Za položen kolokvijum neophodno je sakupiti više od 50% poena na svakom od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

STUDIJSKI PROGRAM ENERGETIKA, ELEKTRONIKA I TELEKOMUNIKACIJE		
K TET Katedra za teorijsku elektrotehniku www.ktet.ftn.uns.ac.rs	Osnovi elektrotehnike 1 (II kolokvijum)	**K2** **16.09.2017.**

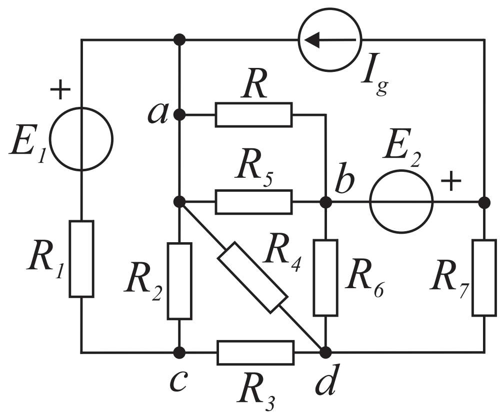
ZADACI

Zadatak 1. U mreži sa slike 1 poznate su brojne vrednosti parametara svih elemenata, osim otpornosti otpornika R.

a) Primenjujući teoremu o kompenzaciji odrediti otpornost otpornika R tako da napon između njegovih priključaka ima vrednost $U_{ab} = 3\ V$.

b) Izračunati snage svih generatora kada otpornost otpornika R ima vrednost izračunatu pod *a*).

Brojni podaci su: $E_1 = 3\ V$, $E_2 = 6\ V$, $I_{g1} = 0{,}15\ A$, $R_1 = 18\ \Omega$, $R_2 = R_5 = R_7 = 36\ \Omega$, $R_3 = 12\ \Omega$, $R_4 = 24\ \Omega$, $R_6 = 72\ \Omega$.



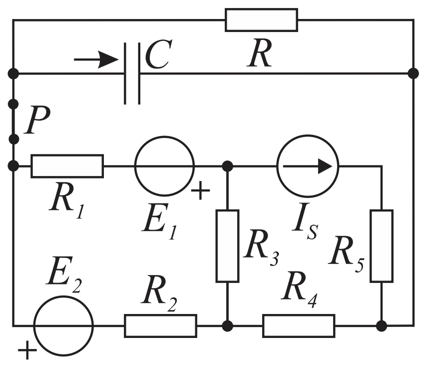
Slika 1

Zadatak 2. U kolu vremenski konstantne struje sa slike 2 nalazi se kondenzator kapacitivnosti C, kroz koji, pri otvaranju prekidača P, protekne količina naelektrisanja $q = 2{,}8\ \mu C$, u naznačenom referentnom smeru.

a) Primenjujući Tevenenovu teoremu, odrediti vrednost kapacitivnosti kondenzatora, C.

b) Odrediti snagu otpornika otpornosti R, kada je prekidač zatvoren.

Brojni podaci su: $R = 6\ \Omega$, $R_1 = R_3 = 10\ \Omega$, $R_2 = R_4 = 30\ \Omega$, $R_5 = 20\ \Omega$, $E_1 = 12\ V$, $E_2 = 28\ V$, $I_S = 1\ A$.



Slika 2.

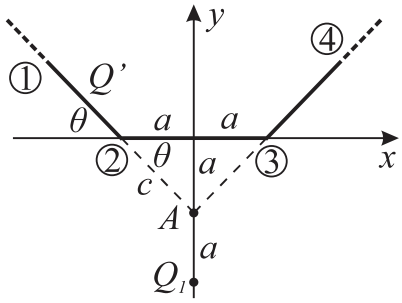
PRAVILA POLAGANJA

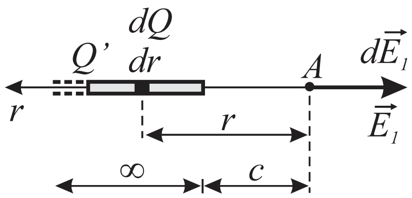
Za položen kolokvijum neophodno je sakupiti više od 50% poena na svakom od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

Ispit iz OET1, 16.09.2017.

I-1

a)

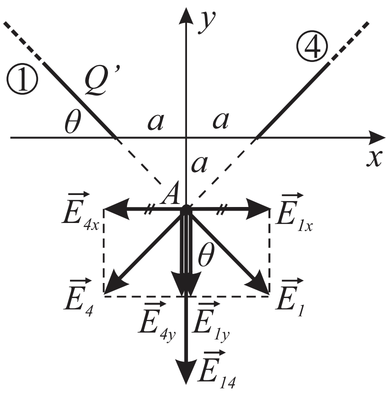




$$c = a\sqrt{2}$$

$$dE_1 = \frac{dQ}{4\pi\varepsilon_0 r^2} = \frac{Q'\,dr}{4\pi\varepsilon_0 r^2}$$

$$E_1 = \int_{\substack{po \\ štapu}} dE_1 = \frac{Q'}{4\pi\varepsilon_0}\int_c^{\infty}\frac{dr}{r^2} = \frac{Q'}{4\pi\varepsilon_0}\left(\frac{1}{a\sqrt{2}} - \frac{1}{\infty}\right) = \frac{Q'}{4\pi\varepsilon_0 a\sqrt{2}} = E_4$$

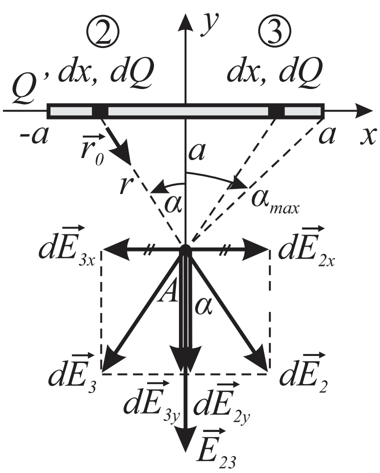


$$\vec{E}_{1x} + \vec{E}_{4x} = 0$$

$$E_{1y} = E_1\cos\theta = \frac{Q'}{4\pi\varepsilon_0 a\sqrt{2}}\frac{\sqrt{2}}{2} = \frac{Q'}{8\pi\varepsilon_0 a} \qquad \boxed{\vec{E}_{1y} = E_{1y}\cdot\left(-\vec{i}_y\right)}$$

$$E_{4y} = E_4\cos\theta = \frac{Q'}{4\pi\varepsilon_0 a\sqrt{2}}\frac{\sqrt{2}}{2} = \frac{Q'}{8\pi\varepsilon_0 a} \qquad \boxed{\vec{E}_{4y} = E_{4y}\cdot\left(-\vec{i}_y\right)}$$

$$\vec{E}_{14} = \vec{E}_{1y} + \vec{E}_{4y} \qquad \boxed{\vec{E}_{14} = \frac{Q'}{4\pi\varepsilon_0 a}\cdot\left(-\vec{i}_y\right)}$$



$$dE_2 = \frac{dQ}{4\pi\varepsilon_0 r^2} = \frac{Q'\,dx}{4\pi\varepsilon_0 r^2}$$

$$d\vec{E}_{2x} + d\vec{E}_{3x} = 0$$

$$dE_{2y} = dE_2\cos\alpha$$

$$dE_{23y} = dE_{2y} + dE_{3y} = 2\,dE_2\cos\alpha = 2\frac{Q'\,dx}{4\pi\varepsilon_0 r^2}\cos\alpha$$

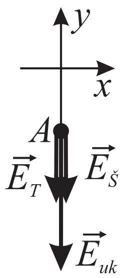
$$dE_{23y} = 2\frac{Q'\dfrac{r\,d\alpha}{\cos\alpha}}{4\pi\varepsilon_0 r^2}\cos\alpha = 2\frac{Q'}{4\pi\varepsilon_0\dfrac{a}{\cos\alpha}}d\alpha = 2\frac{Q'}{4\pi\varepsilon_0 a}\cos\alpha\,d\alpha$$

$$E_{23} = \int_{\substack{po \\ štapu}} dE_{23y} = 2\frac{Q'}{4\pi\varepsilon_0 a}\int_0^{\alpha_{\max}}\cos\alpha\,d\alpha = \frac{Q'}{2\pi\varepsilon_0 a}\left(\sin\frac{\pi}{4} - \sin 0\right) = \frac{Q'}{2\pi\varepsilon_0 a}\frac{\sqrt{2}}{2}$$

$$\boxed{\vec{E}_{23} = \frac{Q'}{2\pi\varepsilon_0 a}\frac{\sqrt{2}}{2}\cdot\left(-\vec{i}_y\right)}$$

$$\vec{E}_{\check{S}} = \vec{E}_{14} + \vec{E}_{23} = \frac{Q'}{4\pi\varepsilon_0 a}\cdot\left(-\vec{i}_y\right) + \frac{Q'}{2\pi\varepsilon_0 a}\frac{\sqrt{2}}{2}\cdot\left(-\vec{i}_y\right) \qquad \boxed{\vec{E}_{\check{S}} = \frac{Q'}{4\pi\varepsilon_0 a}\left(1+\sqrt{2}\right)\cdot\left(-\vec{i}_y\right)}$$

b)



$$E_T = \frac{|Q_1|}{4\pi\varepsilon_0 a^2} \qquad Q_1 < 0 \qquad \boxed{\vec{E}_T = \frac{Q_1}{4\pi\varepsilon_0 a^2}\cdot\left(-\vec{i}_y\right)}$$

$$\left.\begin{aligned} E_{uk} &= E_Š + E_T = \frac{Q'}{4\pi\varepsilon_0 a}\left(1+\sqrt{2}\right) + \frac{|Q_1|}{4\pi\varepsilon_0 a^2} \\ E_{uk} &= 2E_Š \\ 2E_Š &= E_Š + E_T \end{aligned}\right\} \Rightarrow \qquad E_Š = E_T$$

$$\frac{Q'}{4\pi\varepsilon_0 a}\left(1+\sqrt{2}\right) = \frac{|Q_1|}{4\pi\varepsilon_0 a^2}$$

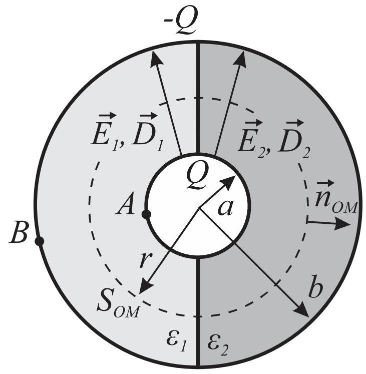
$$|Q_1| = Q'\,a\left(1+\sqrt{2}\right) = 10\cdot 10^{-9}\cdot 1\cdot 10^{-2}\cdot\left(1+\sqrt{2}\right) = 241,42\ pC$$

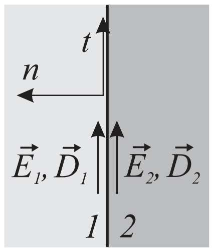
$$\boxed{Q_1 = -241,42\ pC}$$

Ispit iz OET1, 16.09.2017.

I-2

a)





Granični uslov:

$$E_{t1} = E_{t2} \qquad E_1 = E_2 = E$$
$$D_{n1} \neq D_{n2}$$

$$D_1 = \varepsilon_1 \, E$$
$$D_2 = \varepsilon_2 \, E$$

$$\oint_S \vec{D} \cdot \vec{ds} = Q_{slobodno\ u\ S}$$

$$\int_{S_{B1}} \cancel{\vec{D} \cdot \vec{ds}}^{\,0} + \int_{S_{B2}} \cancel{\vec{D} \cdot \vec{ds}}^{\,0} + \int_{S_{OM}} \vec{D} \cdot \vec{ds} = Q_{slobodno\ u\ S}$$

$$\sphericalangle\left(\vec{D}, \vec{n}_{B1}\right) = 90^\circ \qquad \sphericalangle\left(\vec{D}, \vec{n}_{B2}\right) = 90^\circ \qquad \sphericalangle\left(\vec{D}, \vec{n}_{OM}\right) = 0^\circ$$

$$\int_{S_{OM1}} D_1 \, ds + \int_{S_{OM2}} D_2 \, ds = Q$$

$$D_1 \, r\pi L + D_2 \, r\pi L = Q$$

$$\varepsilon_1 \, E \, r\pi L + \varepsilon_2 \, E \, r\pi L = Q$$

$$\boxed{E = \frac{Q}{\varepsilon_1 \, r\pi L + \varepsilon_2 \, r\pi L} = \frac{Q}{\left(\varepsilon_1 + \varepsilon_2\right) r\pi L}}, \qquad a < r < b$$

$$\boxed{D_1 = \varepsilon_1 \, E = \varepsilon_1 \, \frac{Q}{\left(\varepsilon_1 + \varepsilon_2\right) r\pi L}}, \qquad a < r < b$$

$$\boxed{D_2 = \varepsilon_2 \, E = \varepsilon_2 \, \frac{Q}{\left(\varepsilon_1 + \varepsilon_2\right) r\pi L}}, \qquad a < r < b$$

b)

$$U_{AB} = \int_A^B \vec{E} \cdot \vec{dl} = \int_a^b E \, dr = \int_a^b \frac{Q}{\left(\varepsilon_1 + \varepsilon_2\right) r\pi L} \, dr = \frac{Q}{\left(\varepsilon_1 + \varepsilon_2\right) \pi L} \ln \frac{b}{a}$$

$$C = \frac{Q}{U_{AB}} \qquad \boxed{C = \frac{\left(\varepsilon_1 + \varepsilon_2\right) \pi L}{\ln \frac{b}{a}}} \qquad \boxed{C = 1196{,}8 \ pF}$$

c)

$$E_{\max 1} = E_{\max 2} = E_{\max(r=a)} = \frac{Q_{\max}}{\left(\varepsilon_1 + \varepsilon_2\right) a\pi L} = \min\left\{E_{\check{C}1}, E_{\check{C}2}\right\} = E_{\check{C}1} = 65 \ \frac{kV}{cm}$$

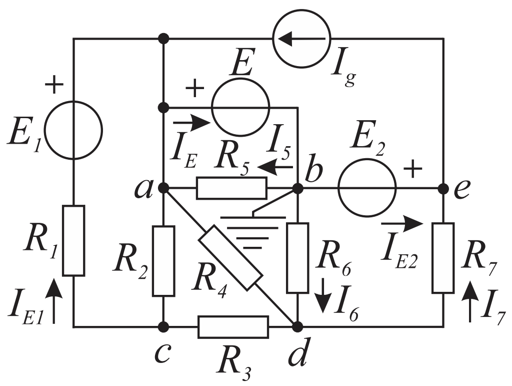
$$\frac{C \, U_{\max}}{\left(\varepsilon_1 + \varepsilon_2\right) a\pi L} = E_{\check{C}1}$$

$$U_{\max} = \frac{E_{\check{C}1} \left(\varepsilon_1 + \varepsilon_2\right) a\pi L}{C} = \frac{E_{\check{C}1} \left(\varepsilon_1 + \varepsilon_2\right) a\pi L}{\dfrac{\left(\varepsilon_1 + \varepsilon_2\right) \pi L}{\ln \frac{b}{a}}} = E_{\check{C}1} \, a \ln \frac{b}{a} \qquad \boxed{U_{\max} = 9{,}96 \ kV}$$

Ispit iz OET1, 16.09.2017.

II-1

a)



$$V_b = 0\,V, \qquad V_a = E = U_{ab} = 3\,V, \qquad V_e = E_2 = 6\,V$$

$$V_c\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right) - V_a\left(\frac{1}{R_1}+\frac{1}{R_2}\right) - V_d\left(\frac{1}{R_3}\right) = -\frac{E_1}{R_1}$$

$$V_d\left(\frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_6}+\frac{1}{R_7}\right) - V_a\left(\frac{1}{R_4}\right) - V_c\left(\frac{1}{R_3}\right) - V_e\left(\frac{1}{R_7}\right) = 0$$

$$V_c\left(\frac{1}{18}+\frac{1}{36}+\frac{1}{12}\right) - 3\cdot\left(\frac{1}{18}+\frac{1}{36}\right) - V_d\left(\frac{1}{12}\right) = -\frac{3}{18} \qquad /\cdot 36$$

$$V_d\left(\frac{1}{12}+\frac{1}{24}+\frac{1}{72}+\frac{1}{36}\right) - 3\cdot\left(\frac{1}{24}\right) - V_c\left(\frac{1}{12}\right) - 6\cdot\left(\frac{1}{36}\right) = 0 \qquad /\cdot 72$$

$$\left.\begin{array}{l} 6V_c - 3V_d = 3 \\ 12V_d - 6V_c = 21 \end{array}\right\} \quad \Rightarrow \quad V_c = 1{,}83\,V, \qquad V_d = 2{,}67\,V$$

$$I_7 = \frac{V_d - V_e}{R_7} = \frac{2{,}67-6}{36} = -0{,}0925\ A$$

$$I_g - I_{E2} - I_7 = 0 \qquad \Rightarrow \qquad I_{E2} = I_g - I_7 = 0{,}15 + 0{,}0925 = 0{,}2425\ A$$

$$I_6 = \frac{0 - V_d}{R_6} = \frac{-2{,}67}{72} = -0{,}037\ A$$

$$I_5 = \frac{0 - V_a}{R_5} = \frac{-3}{36} = -0{,}083\ A$$

$$-I_E + I_5 + I_6 + I_{E2} = 0 \qquad \Rightarrow \qquad I_E = I_5 + I_6 + I_{E2} = -0{,}083 - 0{,}037 + 0{,}2425 = 0{,}1225\ A$$

$$R = \frac{U_{ab}}{I_E} = \frac{3}{0{,}1225} \qquad \boxed{R = 24{,}5\,\Omega}$$

b)

$$I_{E1} = \frac{V_c - V_a + E_1}{R_1} = \frac{1{,}83 - 3 + 3}{18} = 0{,}1017\ A$$

$$P_{E1} = E_1 I_{E1} = 3\cdot 0{,}1017 \qquad \boxed{P_{E1} = 0{,}305\,W}$$

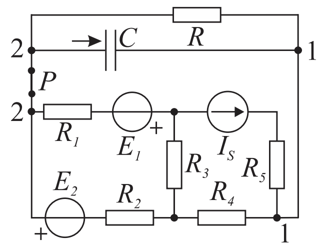
$$P_{E2} = E_2 I_{E2} = 6\cdot 0{,}2425 \qquad \boxed{P_{E2} = 1{,}455\,W}$$

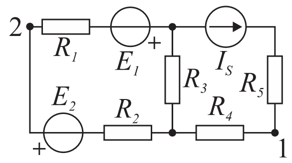
$$P_{Ig} = U_g I_g = \left(V_a - V_e\right) I_g = \left(3-6\right)\cdot 0{,}15 \qquad \boxed{P_{Ig} = -0{,}45\,W}$$

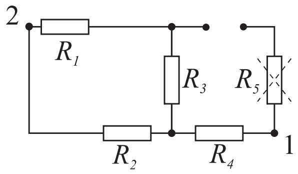
<u>Ispit iz OET1, 16.09.2017.</u>

II-2

a)

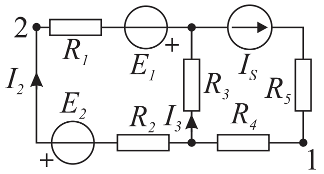






$$R_T = R_{12} = R_4 + \left[\left(R_3 + R_1\right) \| R_2\right] = 30 + \left[\left(10+10\right) \| 30\right] = 30 + \left[20 \| 30\right] = 42$$

$$\boxed{R_T = 42\,\Omega}$$



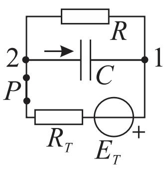
$$R_2 I_2 - E_2 + R_1 I_2 - E_1 - R_3 I_3 = 0$$

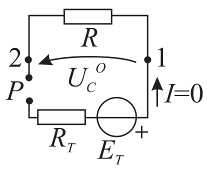
$$I_3 = I_S - I_2$$

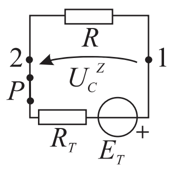
$$R_2 I_2 - E_2 + R_1 I_2 - E_1 - R_3\left(I_S - I_2\right) = 0$$

$$I_2 = \frac{E_1 + E_2 + R_3 I_S}{R_2 + R_1 + R_3} = \frac{12 + 28 + 10 \cdot 1}{30 + 10 + 10} = 1\,A$$

$$E_T = U_{12} = R_4 I_S + R_2 I_2 - E_2 = 30 \cdot 1 + 30 \cdot 1 - 28 \qquad \boxed{E_T = 32\,V}$$







$$q = C\left(U_C^{\,KR} - U_C^{\,PO\check{C}}\right) = C\left(U_C^{\,O} - U_C^{\,Z}\right)$$

$$C = \frac{q}{U_C^{\,O} - U_C^{\,Z}}$$

$$U_C^{\,O} = -R\cancel{I}^{\,0} = 0\,V$$

$$U_C^{\,Z} = -\frac{R}{R + R_T} E_T = -\frac{6}{6 + 42} \cdot 32 = -4\,V$$

$$C = \frac{q}{U_C^{\,O} - U_C^{\,Z}} = \frac{2{,}8 \cdot 10^{-6}}{0 - \left(-4\right)} = \frac{2{,}8 \cdot 10^{-6}}{4} = 0{,}7 \cdot 10^{-6}\,F$$

$$\boxed{C = 0{,}7\,\mu F}$$

b)

Kada je prekidač zatvoren:

$$U_R = U_C^{\,Z} = -4\,V$$

$$P_R^{\,Z} = \frac{U_R^{\,2}}{R} = \frac{\left(-4\right)^2}{6} \qquad \Rightarrow \qquad \boxed{P_R^{\,Z} = 2{,}67\,W}$$