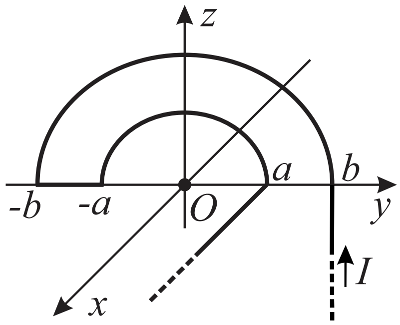
STUDIJSKI PROGRAM ENERGETIKA, ELEKTRONIKA I TELEKOMUNIKACIJE		
K TET Katedra za teorijsku elektrotehniku www.ktet.ftn.uns.ac.rs	Osnovi elektrotehnike 2 (I kolokvijum)	K1 13.09.2017.

ZADACI

Zadatak 1. Veoma dugačak žičani provodnik, sa vremenski konstantnom strujom jačine I, savijen je kao što je prikazano na slici 1. Provodnik se sastoji od tri pravolinijska i dva lučna segmenta, pri čemu jedan pravolinijski segment leži u y-z ravni Dekartovog pravouglog koordinatnog sistema, paralelno sa z osom, drugi leži na y osi, dok je treći u x-y ravni, paralelno sa x osom. Lučni segmenti, u obliku polovine kruga, poluprečnika a i b, leže u y-z ravni. Odrediti intenzitet vektora magnetske indukcije u koordinatnom početku, tačka O. Sredina je vazduh.

Brojne vrednosti: $I = 1\ A$, $a = 2\ cm$, $b = 3\ cm$.

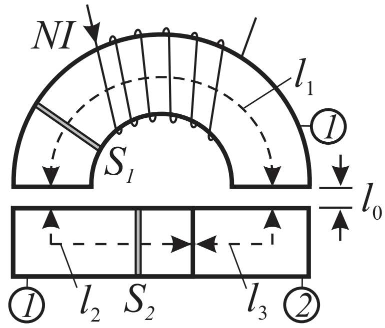


Slika 1.

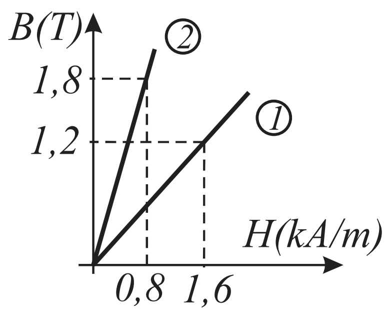
Zadatak 2. Na slici 2a je prikazano magnetsko kolo elektromagneta sa kotvom. Jezgro elektromagneta je načinjeno od materijala 1, dok je kotva načinjena od materijala 1 i 2. Idealizovane krive magnetisanja ovih materijala date su na slici 2b. Da bi elektromagnet privukao kotvu, u procepima treba ostvariti magnetsko polje indukcije $B_0 = 1{,}2\ T$.

a) Odrediti potrebnu vrednost pobude elektromagneta, NI.
b) Izračunati energiju koja se utroši na uspostavljanje magnetskog polja u jezgru elektromagneta i kotvi.

Površina poprečnog preseka jezgra elektromagneta je $S_1 = 2{,}5\ cm^2$, a kotve $S_2 = 2\ cm^2$. Dužine njihovih središnjih linija su $\ell_1 = 30\ cm$, $\ell_2 = 12\ cm$ i $\ell_3 = 8\ cm$. Efektivna površina preseka procepa je $S_0 = 3\ cm^2$. Širina procepa je $\ell_0 = 0{,}1\ mm$.



Slika 2a.



Slika 2b.

PRAVILA POLAGANJA

Za položen kolokvijum neophodno je tačno uraditi više od 50% svakog od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

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ZADACI

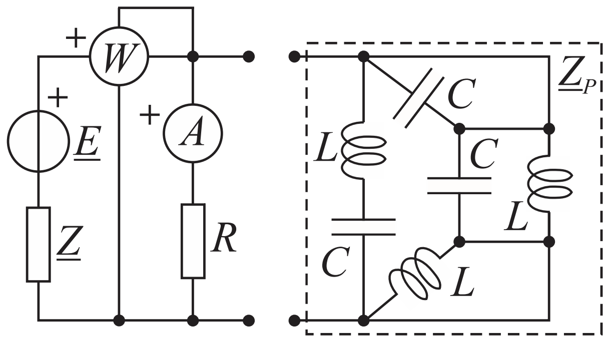
Zadatak 1. U mreži prostoperiodičnih struja prikazanoj na slici 1:

a) Odrediti sve rezonantne i antirezonantne kružne učestanosti prijemnika impedanse $\underline{Z}_P$.

b) Ako u kolu naponski generator *ems* $\underline{E}$ radi na srednjoj rezonantnoj učestanosti, odrediti pokazivanje idealnih mernih instrumenata.

c) Odrediti reaktivnu snagu koja se razvija na naponskom generatoru *ems* $\underline{E}$.

Brojni podaci: $R = 10\ \Omega$, $\underline{Z} = (2 + j1)\ \Omega$, $\underline{E} = j3\ V$, $L = 10\ mH$, $C = 40\ nF$.



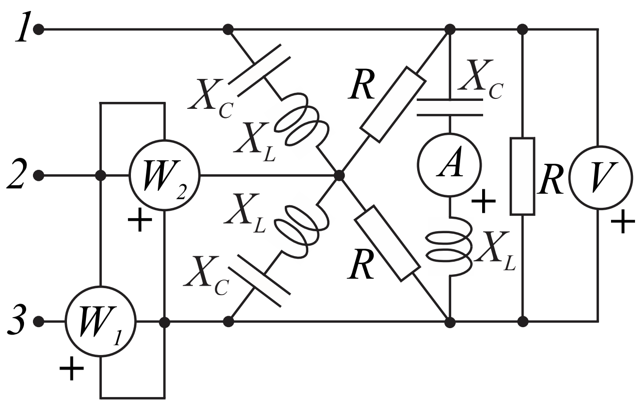
Slika 1.

Zadatak 2. Simetrični trofazni prijemnik, prikazan na slici 2, priključen je na mrežu faznog napona prve faze $U_1 = 230\ V$.

a) Odrediti pokazivanja idealnih mernih instrumenata.

b) Na istom fazorskom dijagramu prikazati fazore faznih napona i svih veličina od kojih zavise pokazivanja idealnih mernih instrumenata.

Brojni podaci: $R = 60\ \Omega$, $X_L = 40\ \Omega$, $X_C = 10\ \Omega$.



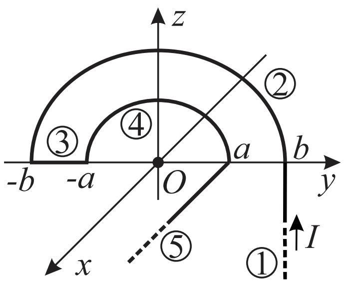
Slika 2.

PRAVILA POLAGANJA

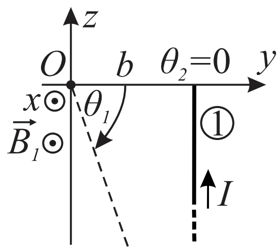
Za položen kolokvijum neophodno je tačno uraditi više od 50% svakog od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

Ispit iz OET2, 13.09.2017.

I-1



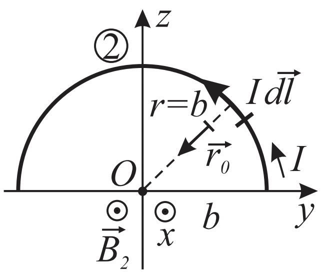
$$\boxed{\vec{B}_3 = 0} \qquad \sin\sphericalangle\left(\vec{dl},\vec{r}_0\right) = 0$$



$\boxed{1}$ $\quad d = b, \quad \theta_1 \to -\dfrac{\pi}{2}, \quad \theta_2 = 0$

$$B_1 = \frac{\mu_0 I}{4\pi d}\left(\sin\theta_2 - \sin\theta_1\right) = \frac{\mu_0 I}{4\pi b}\left[0-(-1)\right] = \frac{\mu_0 I}{4\pi b}$$

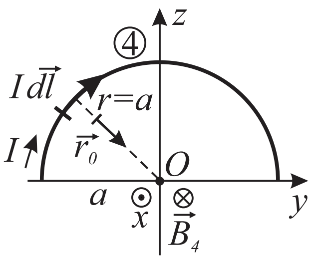
$$\boxed{\vec{B}_1 = \frac{\mu_0 I}{4\pi b}\cdot\vec{i}_x}$$



$$\vec{dB}_2 = \frac{\mu_0}{4\pi}\frac{I\,\vec{dl}\times\vec{r}_0}{r^2} \qquad dB_2 = \frac{\mu_0}{4\pi}\frac{I\,dl}{b^2} \qquad \sphericalangle\left(\vec{dl},\vec{r}_0\right) = \frac{\pi}{2}$$

$$B_2 = \int dB_2 = \frac{\mu_0 I}{4\pi b^2}\int_0^{\frac{1}{2}2b\pi} dl = \frac{\mu_0 I}{4\pi b^2}b\pi = \frac{\mu_0 I}{4b}$$

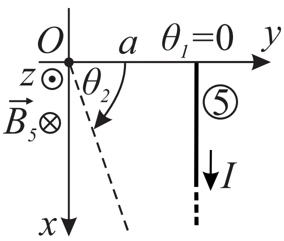
$$\boxed{\vec{B}_2 = \frac{\mu_0 I}{4b}\cdot\vec{i}_x}$$



$$\vec{dB}_4 = \frac{\mu_0}{4\pi}\frac{I\,\vec{dl}\times\vec{r}_0}{r^2} \qquad dB_4 = \frac{\mu_0}{4\pi}\frac{I\,dl}{a^2} \qquad \sphericalangle\left(\vec{dl},\vec{r}_0\right) = \frac{\pi}{2}$$

$$B_4 = \int dB_4 = \frac{\mu_0 I}{4\pi a^2}\int_0^{\frac{1}{2}2a\pi} dl = \frac{\mu_0 I}{4\pi a^2}a\pi = \frac{\mu_0 I}{4a}$$

$$\boxed{\vec{B}_4 = \frac{\mu_0 I}{4a}\cdot\left(-\vec{i}_x\right)}$$



$\boxed{5}$ $\quad d = a, \quad \theta_1 = 0, \quad \theta_2 \to \dfrac{\pi}{2}$

$$B_5 = \frac{\mu_0 I}{4\pi d}\left(\sin\theta_2 - \sin\theta_1\right) = \frac{\mu_0 I}{4\pi a}(1-0) = \frac{\mu_0 I}{4\pi a}$$

$$\boxed{\vec{B}_5 = \frac{\mu_0 I}{4\pi a}\cdot\left(-\vec{i}_z\right)}$$

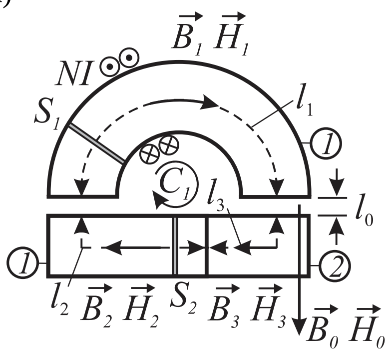
$$\boxed{\begin{aligned}\vec{B}_O &= \vec{B}_1 + \vec{B}_2 + \vec{B}_3 + \vec{B}_4 + \vec{B}_5 = \left(\frac{\mu_0 I}{4\pi b} + \frac{\mu_0 I}{4b} - \frac{\mu_0 I}{4a}\right)\cdot\vec{i}_x + \frac{\mu_0 I}{4\pi a}\cdot\left(-\vec{i}_z\right)\\ \vec{B}_O &= 1{,}9\ \mu T\cdot\left(-\vec{i}_x\right) + 5\ \mu T\cdot\left(-\vec{i}_z\right)\end{aligned}}$$

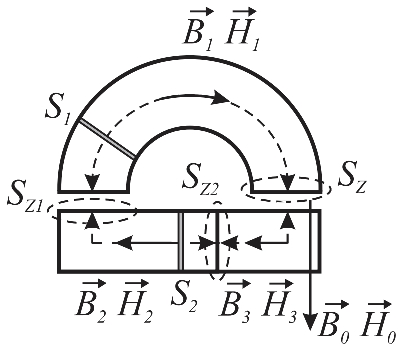
$$\left|\vec{B}_O\right| = \sqrt{\left(1{,}9\ \mu T\right)^2 + \left(5\ \mu T\right)^2} \qquad \boxed{\left|\vec{B}_O\right| = 5{,}35\ \mu T}$$

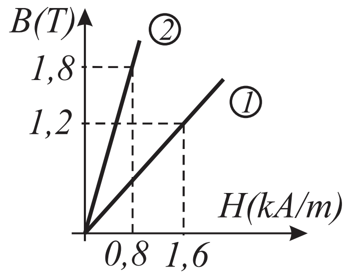
Ispit iz OET2, 13.09.2017.

I-2

a)







$$\oint_{S_Z} \vec{B}\cdot\vec{ds} = 0$$

$S_Z: \quad -B_1 S_1 + B_0 S_0 = 0 \qquad (1)$

$S_{Z1}: \quad -B_2 S_2 + B_0 S_0 = 0 \qquad (2)$

$S_{Z2}: \quad -B_3 S_2 + B_2 S_2 = 0$

$B_3 = B_2 \qquad (3)$

$$\oint_C \vec{H}\cdot\vec{dl} = NI$$

$C_1: \quad H_1 l_1 + 2H_0 l_0 + H_3 l_3 + H_2 l_2 = NI \qquad (4)$

$H_0 = \dfrac{B_0}{\mu_0} \qquad (5)$

Iz uslova zadatka sledi $B_0 = 1,2\,T, \quad \Rightarrow \quad H_0 = \dfrac{B_0}{\mu_0} = \dfrac{1,2}{4\pi\cdot 10^{-7}} = 955,41\,kA/m$

Iz (1) sledi $B_1 = B_0 \dfrac{S_0}{S_1} = 1,2\cdot\dfrac{3\cdot 10^{-4}}{2,5\cdot 10^{-4}} = 1,44\,T$

Sa krive magnetisanja sledi $H_1 = \dfrac{B_1}{\mu_1} = \dfrac{1,44}{\frac{1,2}{1600}} = 1920\,A/m$

Iz (2) sledi $B_2 = B_0 \dfrac{S_0}{S_2} = 1,2\cdot\dfrac{3\cdot 10^{-4}}{2\cdot 10^{-4}} = 1,8\,T$

Sa krive magnetisanja sledi $H_2 = \dfrac{B_2}{\mu_1} = \dfrac{1,8}{\frac{1,2}{1600}} = 2400\,A/m$

Iz (3) sledi $B_3 = B_2 = 1,8\,T$

Sa krive magnetisanja sledi $H_3 = \dfrac{B_3}{\mu_2} = \dfrac{1,8}{\frac{1,8}{800}} = 800\,A/m$

Iz (4) sledi $NI = H_1 l_1 + 2H_0 l_0 + H_3 l_3 + H_2 l_2 = 1920\cdot 0,3 + 2\cdot 955,41\cdot 10^3\cdot 0,1\cdot 10^{-3} + 800\cdot 0,08 + 2400\cdot 0,12$

$\boxed{NI = 1119,1\,Azav}$

b)

$$W_{mJ} = \frac{1}{2}B_1 H_1 V_1 = \frac{1}{2}B_1 H_1 l_1 S_1 = \frac{1}{2}\cdot 1,44\cdot 1920\cdot 0,3\cdot 2,5\cdot 10^{-4} \qquad \boxed{W_{mJ} = 103,68\,mJ}$$

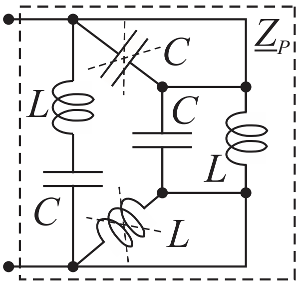
$$W_{mK} = \frac{1}{2}B_2 H_2 V_2 + \frac{1}{2}B_3 H_3 V_3 = \frac{1}{2}B_2 H_2 l_2 S_2 + \frac{1}{2}B_3 H_3 l_3 S_2$$

$$W_{mK} = \frac{1}{2}\cdot 1,8\cdot 2400\cdot 0,12\cdot 2\cdot 10^{-4} + \frac{1}{2}\cdot 1,8\cdot 800\cdot 0,08\cdot 2\cdot 10^{-4} \qquad \boxed{W_{mK} = 63,36\,mJ}$$

Ispit iz OET2, 13.09.2017.

II-1

a)



$$\underline{Z}_1 = j\omega L \parallel \frac{1}{j\omega C} = \frac{j\omega L \dfrac{1}{j\omega C}}{j\omega L + \dfrac{1}{j\omega C}} = \frac{j\omega L}{1-\omega^2 LC}$$

$$\underline{Z}_2 = j\omega L + \frac{1}{j\omega C} = \frac{1-\omega^2 LC}{j\omega C}$$

$$\underline{Z}_P = \underline{Z}_1 \parallel \underline{Z}_2 = \frac{j\omega L}{1-\omega^2 LC} \parallel \frac{1-\omega^2 LC}{j\omega C} = \frac{\dfrac{j\omega L}{1-\omega^2 LC}\,\dfrac{1-\omega^2 LC}{j\omega C}}{\dfrac{j\omega L}{1-\omega^2 LC} + \dfrac{1-\omega^2 LC}{j\omega C}} \qquad / \cdot \frac{\left(1-\omega^2 LC\right) j\omega C}{\left(1-\omega^2 LC\right) j\omega C}$$

$$\underline{Z}_P = \frac{j\omega L\left(1-\omega^2 LC\right)}{j\omega L\ j\omega C + \left(1-\omega^2 LC\right)^2} = \frac{j\omega L\left(1-\omega^2 LC\right)}{-\omega^2 LC + 1 - 2\omega^2 LC + \omega^4 L^2 C^2} \qquad \boxed{\underline{Z}_P = j\frac{\omega L\left(1-\omega^2 LC\right)}{\omega^4 L^2 C^2 - 3\omega^2 LC + 1}}$$

$$\mathrm{Im}\left\{\underline{Z}_P\right\} = 0 \qquad \Rightarrow \qquad \omega L = 0 \qquad \boxed{\omega_{r1} = 0}$$

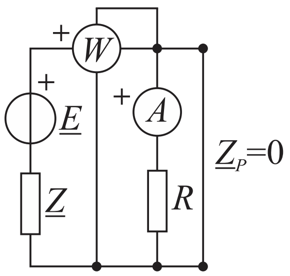
$$1-\omega^2 LC = 0 \qquad \boxed{\omega_{r2} = \frac{1}{\sqrt{LC}}} \qquad \boxed{\omega_{r3} \to \infty}$$

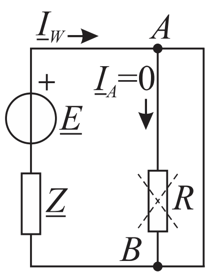
$$\mathrm{Im}\left\{\underline{Y}_P\right\} = 0 \qquad \Rightarrow \qquad \omega^4 L^2 C^2 - 3\omega^2 LC + 1 = 0 \qquad \left(\omega^2 LC = x\right)$$

$$x^2 - 3x + 1 = 0 \qquad \Rightarrow \qquad x_1 = 0{,}38 \qquad x_2 = 2{,}62$$

$$\boxed{\omega_{ar1} = \sqrt{\frac{0{,}38}{LC}}} \qquad \boxed{\omega_{ar2} = \sqrt{\frac{2{,}62}{LC}}}$$

b)





$$\omega = \omega_{r2} = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10\cdot 10^{-3}\cdot 40\cdot 10^{-9}}} = 5\cdot 10^4\ \frac{rad}{s}$$

$$\underline{Z}_P = 0$$

$$\boxed{I_A = \left|\underline{I}_A\right| = 0\ A}$$

$$\underline{I}_W = \frac{\underline{E}}{\underline{Z}} = \frac{j3}{2+j1} = \frac{j3}{2+j1}\,\frac{2-j1}{2-j1} = \frac{3+j6}{5} = \left(0{,}6 + j1{,}2\right)A$$

$$\underline{U}_W = \underline{U}_{AB} = 0$$

$$\underline{S}_W = \underline{U}_W\,\underline{I}_W^{\,*} = 0\,VA$$

$$P_W = \mathrm{Re}\left\{\underline{S}_W\right\} \qquad \Rightarrow \qquad \boxed{P_W = 0\,W}$$

c)

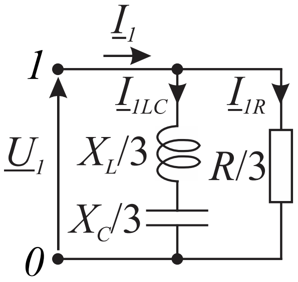
$$\underline{S}_E = \underline{E}\,\underline{I}_W^{\,*} = j3\cdot\left(0{,}6 - j1{,}2\right) = \left(3{,}6 + j1{,}8\right)VA$$

$$Q_E = \mathrm{Im}\left\{\underline{S}_E\right\} \qquad \Rightarrow \qquad \boxed{Q_E = 1{,}8\,VAr}$$

Ispit iz OET2, 13.09.2017.

II-2

a)



$$\underline{I}_{1R} = \frac{\underline{U}_1}{\frac{R}{3}} = \frac{230\, e^{j0°}}{\frac{60}{3}} = 11{,}5\, e^{j0°}\ A = 11{,}5\ A$$

$$\underline{I}_{1LC} = \frac{\underline{U}_1}{j\frac{X_L}{3} - j\frac{X_C}{3}} = \frac{230\, e^{j0°}}{j\frac{40}{3} - j\frac{10}{3}} = \frac{230\, e^{j0°}}{j10} = \frac{230\, e^{j0°}}{10\, e^{j90°}} = 23\, e^{-j90°}\ A = -j23\ A$$

$$\underline{I}_1 = \underline{I}_{1R} + \underline{I}_{1LC} = (11{,}5 - j23)\ A = 25{,}71\, e^{j-63{,}43}\ A$$

$$\underline{I}_2 = \underline{I}_1\, e^{-j120°} = 25{,}71\, e^{-j183{,}43°}\ A$$

$$\underline{I}_3 = \underline{I}_1\, e^{-j240°} = 25{,}71\, e^{-j303{,}43°}\ A$$

$$\underline{I}_A = \frac{\underline{U}_{31}}{jX_L - jX_C} = \frac{\sqrt{3}\cdot 230\, e^{-j210°}}{j40 - j10} = \frac{\sqrt{3}\cdot 230\, e^{-j210°}}{j30} = \frac{\sqrt{3}\cdot 230\, e^{-j210°}}{30\, e^{j90°}} = 13{,}28\, e^{-j300°}\ A$$

$$\boxed{I_A = |\underline{I}_A| = 13{,}28\ A}$$

$$\boxed{U_V = |\underline{U}_{31}| = \sqrt{3}\,|\underline{U}_1| = \sqrt{3}\cdot 230 = 398{,}37\ V}$$

$$P_{W1} = \mathrm{Re}\left\{\underline{U}_{32}\, \underline{I}_3^{\,*}\right\} = U_{32}\, I_3 \cos\sphericalangle(\underline{U}_{32}, \underline{I}_3) = \sqrt{3}\,U_1\, I_1 \cos(303{,}43° - 270°) = \sqrt{3}\cdot 230\cdot 25{,}71\cdot\cos 33{,}43°$$

$$\boxed{P_{W1} = 8547{,}67\ W}$$

$$P_{W2} = \mathrm{Re}\left\{\underline{U}_{32}\, \underline{I}_2^{\,*}\right\} = U_{32}\, I_2 \cos\sphericalangle(\underline{U}_{32}, \underline{I}_2) = \sqrt{3}\,U_1\, I_1 \cos(270° - 183{,}43°) = \sqrt{3}\cdot 230\cdot 25{,}71\cdot\cos 86{,}57°$$

$$\boxed{P_{W2} = 612{,}78\ W}$$

b)