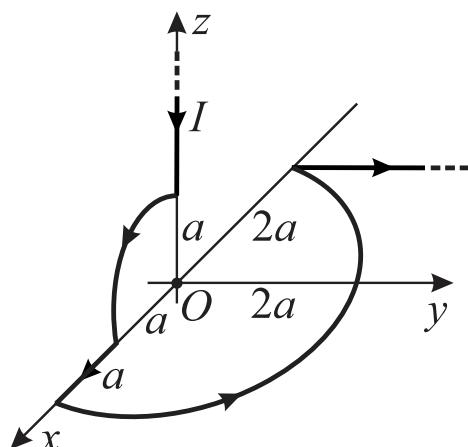


## ZADACI

**Zadatak 1.** Veoma dugačak žičani provodnik, sa vremenski konstantnom strujom jačine  $I$ , savijen je kao što je prikazano na slici 1. Provodnik se sastoji od dva veoma dugačka, pravolinijska i dva lučna segmenta, pri čemu jedan pravolinijski segment leži na  $z$  osi, dok je drugi u  $x$ - $y$  ravni, paralelno sa  $y$  osom, Dekartovog pravouglog koordinatnog sistema. Jedan lučni segment je u obliku četvrtine kruga, poluprečnika  $a$ , i leži u  $x$ - $z$  ravni, dok je drugi u obliku polovine kruga, poluprečnika  $2a$ , i leži u  $x$ - $y$  ravni. Odrediti intenzitet vektora magnetske indukcije u koordinatnom početku. Sredina je vazduh.

Brojne vrednosti:  $I = 1/\pi A$ ,  $a = 1 \text{ cm}$ .

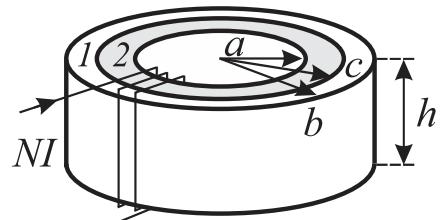


Slika 1.

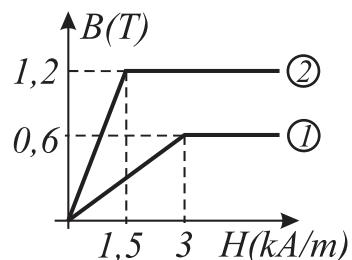
**Zadatak 2.** Ne debelom torusnom jezgru načinjenom od dva feromagnetska materijala, prikazanom na slici 2a, nalazi se namotaj sa  $N = 314$  zavojaka tanke, provodne žice, namotanih ravnomođno i gusto po celom torusu. Krive prvobitnog magnetisanja oba materijala su prikazane na slici 2b.

- Izračunati minimalnu vrednost jačine struje u namotajima,  $I$ , tako da oba sloja jezgra budu u zasićenju.
- Odrediti nove režime rada, oba sloja jezgra, nakon smanjenja struje određene pod a) na duplo manju vrednost.
- Odrediti energiju sadržanu u jezgru nakon smanjenja struje određene pod a) na duplo manju vrednost.

Dimenzije jezgra su  $a = 4 \text{ cm}$ ,  $c = 6 \text{ cm}$ ,  $b = 8 \text{ cm}$ ,  $h = 3 \text{ cm}$ .



Slika 2a.



Slika 2b.

## PRAVILA POLAGANJA

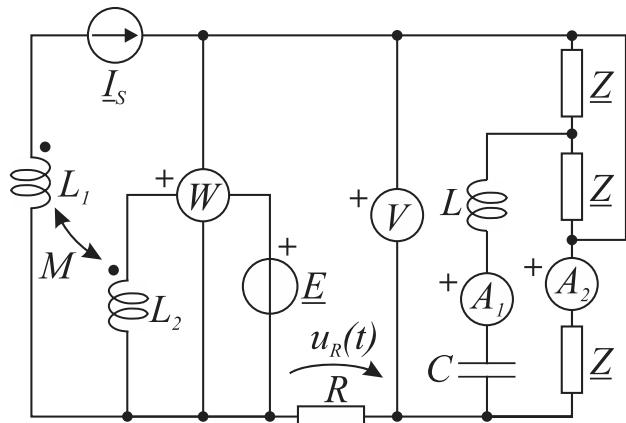
Za položen kolokvijum neophodno je tačno uraditi više od 50% svakog od zadatka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

## ZADACI

**Zadatak 1.** U mreži prostoperiodičnih struja prikazanoj na slici 1:

- Odrediti pokazivanja idealnih mernih instrumenata.
- Napisati izraz za trenutnu vrednost napona na otporniku  $R$ ,  $u_R(t)$ .
- Odrediti reaktivnu snagu koja se razvija na naponskom generatoru ems  $\underline{E}$ .

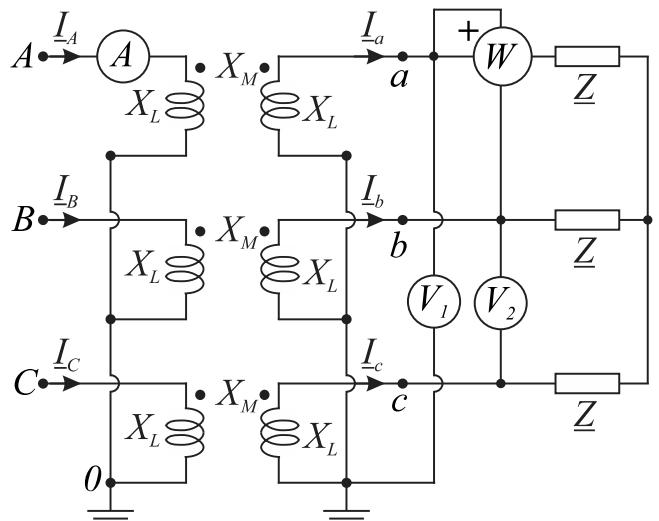
Brojni podaci:  $X_L = X_C = 5 \Omega$ ,  $X_{L1} = 2X_{L2} = 20 \Omega$ ,  $X_M = 2 \Omega$ ,  $R = 4 \Omega$ ,  $\underline{Z} = (3 + j3) \Omega$ ,  $\underline{I}_S = (-2 + j5) A$ ,  $\underline{E} = j4 V$ ,  $\omega = 314 rad/s$ .



Slika 1.

**Zadatak 2.** Trofazni prijemnik, impedansi  $\underline{Z} = (5 - j10) \Omega$ , priključen je na sekundar trofaznog transformatora, kao što je prikazano na slici 2. Primar transformatora je priključen na trofaznu, simetričnu mrežu faznog napona prve faze  $U_A = 230 V$ .

- Izračunati kompleksne jačine struja primara i sekundara kao i kompleksne fazne napone sekundara transformatora.
- Odredite pokazivanja idealnih mernih instrumenata.
- Na istom fazorskom dijagramu prikazati fazore faznih napona sekundara i fazore svih veličina od kojih zavise pokazivanja idealnih mernih instrumenata.



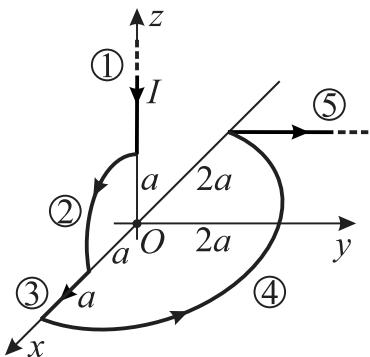
Brojne vrednosti:  $X_L = 10 \Omega$  i  $X_M = 5 \Omega$ .

Slika 2.

## PRAVILA POLAGANJA

Za položen kolokvijum neophodno je tačno uraditi više od 50% svakog od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

I-1



$$\boxed{\vec{B}_1 = 0} \quad \sin \alpha(\vec{dl}, \vec{r}_0) = 0$$

$$\boxed{\vec{B}_3 = 0} \quad \sin \alpha(\vec{dl}, \vec{r}_0) = 0$$

$$\overrightarrow{dB}_2 = \frac{\mu_0}{4\pi} \frac{I \overrightarrow{dl} \times \vec{r}_0}{r^2}$$

$$dB_2 = \frac{\mu_0 I}{4\pi a^2} \int_0^{2a\pi} dl = \frac{\mu_0 I}{4\pi a^2} \frac{1}{2} a\pi = \frac{\mu_0 I}{8a}$$

$$\boxed{\vec{B}_2 = \frac{\mu_0 I}{8a} \cdot \vec{i}_y}$$

$$\overrightarrow{dB}_4 = \frac{\mu_0}{4\pi} \frac{I \overrightarrow{dl} \times \vec{r}_0}{r^2}$$

$$dB_4 = \frac{\mu_0 I}{4\pi (2a)^2} \int_0^{2\cdot 2a\pi} dl = \frac{\mu_0 I}{16\pi a^2} 2a\pi = \frac{\mu_0 I}{8a}$$

$$\boxed{\vec{B}_4 = \frac{\mu_0 I}{8a} \cdot \vec{i}_z}$$

$$\boxed{[5] \quad d = 2a, \quad \theta_1 = 0, \quad \theta_2 \rightarrow \frac{\pi}{2}}$$

$$\vec{B}_5 = \frac{\mu_0 I}{4\pi d} (\sin \theta_2 - \sin \theta_1) = \frac{\mu_0 I}{4\pi 2a} [1 - 0] = \frac{\mu_0 I}{8\pi a}$$

$$\boxed{\vec{B}_5 = \frac{\mu_0 I}{8\pi a} \cdot (-\vec{i}_z)}$$

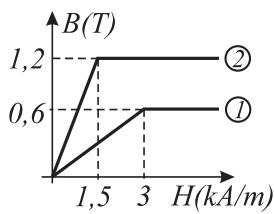
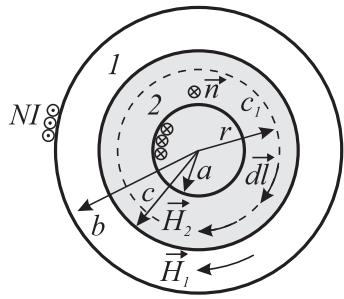
$$\boxed{\vec{B}_o = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 + \vec{B}_4 + \vec{B}_5 = \frac{\mu_0 I}{8a} \cdot \vec{i}_y + \left( \frac{\mu_0 I}{8a} - \frac{\mu_0 I}{8\pi a} \right) \cdot \vec{i}_z}$$

$$\vec{B}_o = 5 \mu T \cdot \vec{i}_y + 3,4 \mu T \cdot \vec{i}_z$$

$$\boxed{|\vec{B}_o| = \sqrt{(5 \mu T)^2 + (3,4 \mu T)^2}}$$

$$\boxed{|\vec{B}_o| = 6,05 \mu T}$$

I-2



$$\text{Granični uslov: } H_{1t} = H_{2t} = H$$

$$\oint_{c1} \vec{H} \cdot d\vec{l} = NI$$

$$H \cdot 2r\pi = NI$$

$$H = \frac{NI}{2r\pi} \quad a < r < b$$

a)

$$H_{1\min}(r=b) = \frac{NI_1}{2\pi b} \geq H_{K1} = 3000 \text{ A/m}$$

$$I_1 = \frac{H_{K1} 2\pi b}{N} = \frac{3000 \cdot 2\pi \cdot 0,08}{314} = 4,8 \text{ A}$$

$$H_{2\min}(r=c) = \frac{NI_2}{2\pi c} \geq H_{K2} = 1500 \text{ A/m}$$

$$I_2 = \frac{H_{K2} 2\pi c}{N} = \frac{1500 \cdot 2\pi \cdot 0,06}{314} = 1,8 \text{ A}$$

$$I = \max\{I_1, I_2\} = I_1$$

$$I = 4,8 \text{ A}$$

b)

$$I^N = \frac{I}{2} = 2,4 \text{ A}$$

$$H_{2\max}(r=a) = \frac{NI^N}{2\pi a} = 3000 \text{ A/m} \geq H_{K2}$$

$\Rightarrow$  Materijal 2 i dalje u zasićenju.

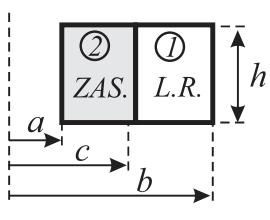
$$H_{2\min}(r=c) = \frac{NI^N}{2\pi c} = 2000 \text{ A/m} \geq H_{K2}$$

$$H_{1\max}(r=c) = \frac{NI^N}{2\pi c} = 2000 \text{ A/m} \leq H_{K1}$$

$\Rightarrow$  Materijal 1 u linearnom režimu rada.

$$H_{1\min}(r=b) = \frac{NI^N}{2\pi b} = 1500 \text{ A/m} \leq H_{K1}$$

c)



$$W_{m2} = \frac{1}{2} B_{K2} H_{K2} V_2 = \frac{1}{2} B_{K2} H_{K2} (c^2 - a^2) \pi h$$

$$W_{m2} = \frac{1}{2} \cdot 1,2 \cdot 1500 \cdot (0,06^2 - 0,04^2) \cdot 3,14 \cdot 0,03$$

$$W_{m2} = 169,6 \text{ mJ}$$

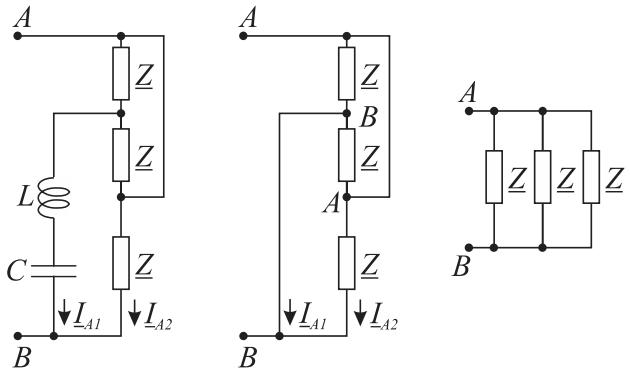
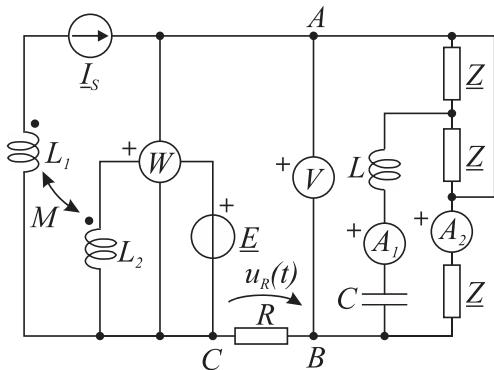
$$W_{m1} = \int_{V_1} \omega_{m1} dV = \int_{V_1} \frac{1}{2} B_1 H_1 dV = \int_{V_1} \frac{1}{2} \mu_1 \left( \frac{NI^N}{2\pi r} \right)^2 2\pi r dr h = \int_{V_1} \frac{1}{2} \mu_1 \frac{(NI^N)^2}{2\pi r} dr h$$

$$W_{m1} = \frac{1}{2} \mu_1 \frac{(NI^N)^2}{2\pi} h \int_c^b \frac{dr}{r} = \frac{1}{2} \mu_1 \frac{(NI^N)^2}{2\pi} h \ln \frac{b}{c} = \frac{1}{2} \frac{0,6}{3000} \frac{(314 \cdot 2,4)^2}{6,28} \cdot 0,03 \cdot \ln \frac{0,08}{0,06}$$

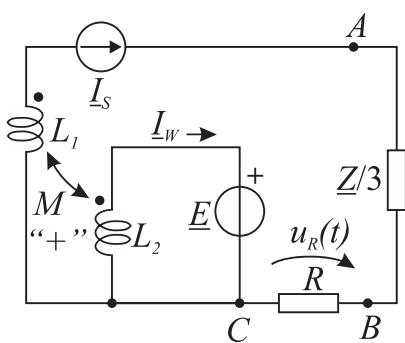
$$W_{m1} = 78 \text{ mJ}$$

$$W_m = W_{m1} + W_{m2} = 247,6 \text{ mJ}$$

II-1



a)



$$jX_{L2} \underline{I}_W + jX_M \underline{I}_s + \underline{E} = 0$$

$$\underline{I}_W = \frac{-\underline{E} - jX_M \underline{I}_s}{jX_{L2}} = \frac{-j4 - j2 \cdot (-2 + j5)}{j10} = \frac{-j4 + j4 + 10}{j10} = -j A$$

$$\underline{U}_W = \underline{U}_{AC} = \underline{I}_s \left( \frac{\underline{Z}}{3} + R \right) = (-2 + j5) \cdot \left( \frac{3 + j3}{3} + 4 \right) = (-2 + j5) \cdot (5 + j)$$

$$\underline{U}_W = -10 - 5 + j25 - j2 = (-15 + j23) V$$

$$\underline{S}_W = \underline{U}_W \underline{I}_W^* = (-15 + j23) \cdot j = (-23 - j15) VA$$

$$P_W = \operatorname{Re}\{\underline{S}_W\} \Rightarrow P_W = -23 W$$

$$\underline{U}_V = \underline{U}_{AB} = \underline{I}_s \frac{\underline{Z}}{3} = (-2 + j5) \cdot \frac{3 + j3}{3} = (-2 + j5) \cdot (1 + j)$$

$$\underline{U}_V = -2 - 5 + j5 - j2 = (-7 + j3) V$$

$$U_V = |\underline{U}_V| = \sqrt{(-7)^2 + 3^2} \Rightarrow U_V = 7,62 V$$

$$\underline{I}_{A2} = \frac{\underline{U}_{AB}}{\underline{Z}} = \frac{\underline{I}_s \frac{\underline{Z}}{3}}{\underline{Z}} = \frac{\underline{I}_s}{3} = \frac{-2 + j5}{3} = \left( -\frac{2}{3} + j\frac{5}{3} \right) A$$

$$I_{A2} = |\underline{I}_{A2}| = \sqrt{\left( -\frac{2}{3} \right)^2 + \left( \frac{5}{3} \right)^2} \Rightarrow I_{A2} = 1,79 A$$

$$\underline{I}_{A1} = \underline{I}_s - \underline{I}_{A2} = \underline{I}_s - \frac{\underline{I}_s}{3} = \frac{2}{3} \underline{I}_s = \frac{2}{3} (-2 + j5) = \left( -\frac{4}{3} + j\frac{10}{3} \right) A$$

$$I_{A1} = |\underline{I}_{A1}| = \sqrt{\left( -\frac{4}{3} \right)^2 + \left( \frac{10}{3} \right)^2} \Rightarrow I_{A1} = 3,59 A$$

b)

$$\underline{U}_R = \underline{U}_{BC} = \underline{I}_s R = (-2 + j5) \cdot 4 = (-8 + j20) V = 21,54 e^{j111,8^\circ} V$$

$$u_R(t) = 21,54 \sqrt{2} \cos(314t + 111,8^\circ) V$$

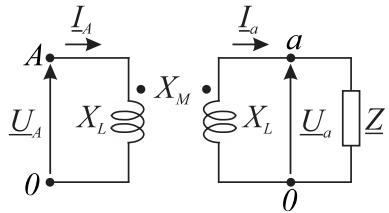
c)

$$\underline{S}_E = -\underline{E} \underline{I}_W^* = -j4 \cdot j = 4 VA$$

$$Q_E = \operatorname{Im}\{\underline{S}_E\} \Rightarrow Q_E = 0 VAr$$

**II-2**

a)



$$(1) \quad \underline{U}_A = jX_L \underline{I}_A - jX_M \underline{I}_a$$

$$(2) \quad \underline{U}_a = -jX_L \underline{I}_a + jX_M \underline{I}_A$$

$$(3) \quad \underline{U}_a = \underline{Z} \underline{I}_a$$

$$(3) \Rightarrow (2) \quad -jX_L \underline{I}_a + jX_M \underline{I}_A = \underline{Z} \underline{I}_a \quad \Rightarrow \quad \underline{I}_a = \frac{jX_M}{\underline{Z} + jX_L} \underline{I}_A = \frac{j5}{(5 - j10) + j10} \underline{I}_A = j1 \underline{I}_A$$

$$(1) \Rightarrow \underline{U}_A = jX_L \underline{I}_A - jX_M \cdot j1 \underline{I}_A = jX_L \underline{I}_A + X_M \underline{I}_A$$

$$\underline{I}_A = \frac{\underline{U}_A}{jX_L + X_M} = \frac{230 e^{j0^\circ}}{5 + j10} = \frac{230 e^{j0^\circ}}{11,18 e^{j63,43^\circ}}$$

$$\boxed{\underline{I}_A = 20,57 e^{-j63,43^\circ} A}$$

$$\boxed{\underline{I}_B = 20,57 e^{-j183,43^\circ} A}$$

$$\boxed{\underline{I}_C = 20,57 e^{-j303,43^\circ} A}$$

$$\underline{I}_a = j1 \underline{I}_A = 1 e^{j90^\circ} \cdot 20,57 e^{-j63,43^\circ}$$

$$\boxed{\underline{I}_a = 20,57 e^{j26,57^\circ} A}$$

$$\boxed{\underline{I}_b = 20,57 e^{-j93,43^\circ} A}$$

$$\boxed{\underline{I}_c = 20,57 e^{-j213,43^\circ} A}$$

$$(3) \Rightarrow \underline{U}_a = \underline{Z} \underline{I}_a = (5 - j10) \cdot 20,57 e^{j26,57^\circ} = 11,18 e^{-j63,43^\circ} \cdot 20,57 e^{j26,57^\circ}$$

$$\boxed{\underline{U}_a = 229,97 e^{-j36,86^\circ} V}$$

$$\boxed{\underline{U}_b = 229,97 e^{-j216,86^\circ} V}$$

$$\boxed{\underline{U}_c = 229,97 e^{-j336,86^\circ} V}$$

b)

$$I_{amp} = |\underline{I}_A| = 20,57 A$$

$$U_{V1} = |\underline{U}_a| = 229,97 V$$

$$U_{V2} = |\underline{U}_{bc}| = \sqrt{3} |\underline{U}_a| = \sqrt{3} \cdot 229,97 = 398,32 V$$

$$P_W = \operatorname{Re} \left\{ \underline{U}_{ab} \underline{I}_a^* \right\} = \underline{U}_{ab} \underline{I}_a \cos \varphi(\underline{U}_{ab}, \underline{I}_a) = \sqrt{3} U_a I_a \cos(63,43^\circ - 30^\circ) = \sqrt{3} \cdot 229,97 \cdot 20,57 \cdot \cos 33,43^\circ$$

$$\boxed{P_W = 6837,91 W}$$

c)

