STUDIJSKI PROGRAM ENERGETIKA, ELEKTRONIKA I TELEKOMUNIKACIJE

K TET
Katedra za teorijsku elektrotehniku
www.ktet.ftn.uns.ac.rs

Osnovi elektrotehnike 2
(I kolokvijum)

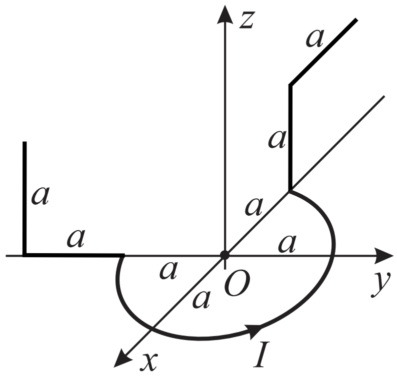
K1 - A

27.06.2017.

ZADACI

Zadatak 1. Žičani provodnik konačne dužine, sa vremenski konstantnom strujom jačine I, savijen je kao što je prikazano na slici 1. Provodnik se sastoji od četiri pravolinijska i jednog lučnog segmenta, kao što je prikazano na slici. Dva pravolinijska segmetna leže u y-z ravni, dok preostala dva u x-z ravni, Dekartovog pravouglog koordinatnog sistema. Lučni segment je u obliku 3/4 kruga, poluprečnika a, i leži u x-y ravni. Odrediti intenzitet vektora magnetske indukcije u koordinatnom početku. Sredina je vazduh.

Brojne vrednosti: $I = 3\ A$, $a = 5\ cm$.

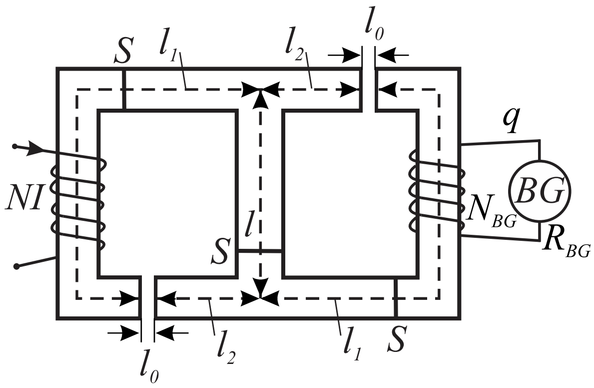


Slika 1.

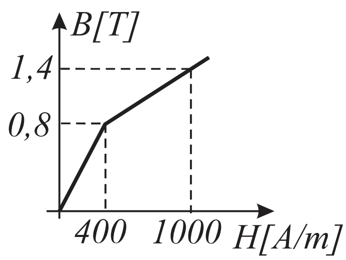
Zadatak 2. Tanko magnetsko kolo sa slike 2a je načinjeno od materijala, čija je idealizovana kriva prvobitnog magnetisanja prikazana na slici 2b.

a) Odrediti smer protekle količine naelektrisanja, q, prilikom uspostavljanja magnetskog polja u jezgru. Pre uključivanja struje, jezgro je bilo nenamagnetisano.
b) Izračunati magnetomotornu pobudnu silu, NI, kako bi, prilikom uspostavljanja magnetskog polja u jezgru, kroz kolo balističkog galvanometra protekla količina naelektrisanja $q = 50\ \mu C$, u smeru određenom pod a). Rasipanja magnetskog fluksa zanemariti.
c) Odrediti energiju utrošenu na uspostavljanje magnetskog polja u grani sa balističkim galvanometrom.

Brojni podaci: $l_0 = 0{,}2\ mm$, $l_1 = 24\ cm$, $l_2 = 6\ cm$, $l = 10\ cm$, $S = 2\ cm^2$, $N_{BG} = 5$ zavojaka, $R_{BG} = 10\ \Omega$.



Slika 2a.



Slika 2b.

PRAVILA POLAGANJA

Za položen kolokvijum neophodno je tačno uraditi više od 50% svakog od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

STUDIJSKI PROGRAM ENERGETIKA, ELEKTRONIKA I TELEKOMUNIKACIJE

K TET Katedra za teorijsku elektrotehniku www.ktet.ftn.uns.ac.rs	# Osnovi elektrotehnike 2 (I kolokvijum)	**K2 - A** **27.06.2017.**

ZADACI

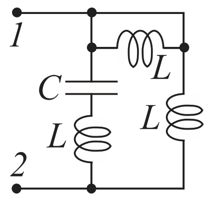
Zadatak 1. Prijemnik sa slike 1a treba da se priključi na ulazni deo kola sa slike 1b, između tačaka A i B.

a) Odrediti sve rezonantne i antirezonantne kružne učestanosti prijemnika prikazanog na slici 1a.

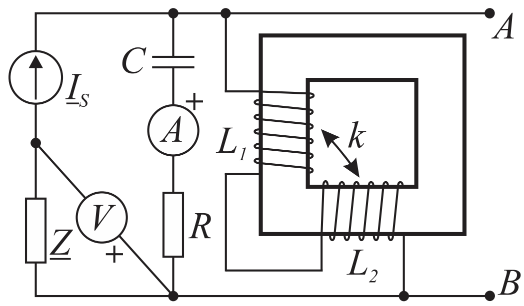
b) Nacrtati električnu šemu ulaznog dela kola i u njoj označiti tip magnetske sprege koji postoji između namotaja induktivnosti L_1 i L_2 (koristiti simbol tačkica).

c) Odrediti pokazivanja idealnih mernih instrumenata, kada je prijemnik priključen na ulazni deo kola, pri čemu je ugaona učestanost struje strujnog generatora $\underline{I}_S$ jednaka $5 \cdot 10^4$ rad/s.

Brojni podaci: $L = 10$ mH, $C = 40$ nF, $R = 30$ Ω, $\underline{Z} = (5 + j3)$ Ω, $\underline{I}_S = (1 - j)$ Ω, $L_1 = 2L$, $L_2 = 3L$, $k = 0{,}7$.



Slika 1a.



Slika 1b.

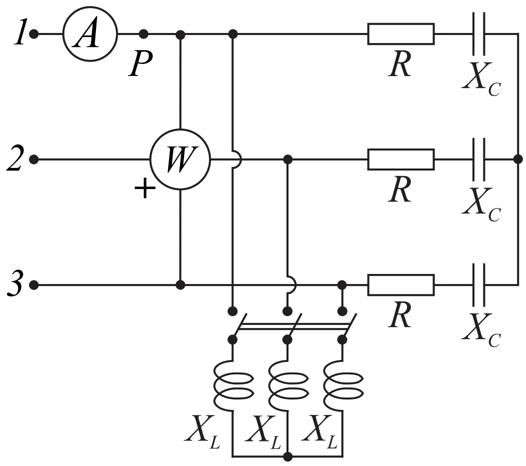
Zadatak 2. Na slici 2 je prikazan simetričan trofazni prijemnik, sastavljen od otpornika otpornosti $R = 60$ Ω i kondenzatora reaktanse $X_C = 180$ Ω. Prijemnik je priključen na mrežu faznog napona $\underline{U}_1 = 230$ V.

a) Odrediti reaktanse kalemova, koje je potrebno povezati, kako bi se faktor snage prijemnika popravio na jedinicu.

b) Odrediti pokazivanja idealnih mernih instrumenata posle priključivanja kalemova.

c) Odrediti pokazivanja idealnih mernih instrumenata, dok su kalemovi priključeni, ali kada se desi prekid provodnika prve faze, u tački P.

d) Na istom fazorskom dijagramu prikazati fazore svih veličina od kojih zavise pokazivanja instrumenata.



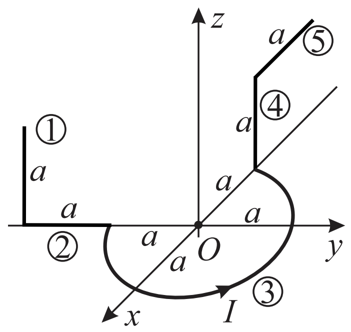
Slika 2.

PRAVILA POLAGANJA

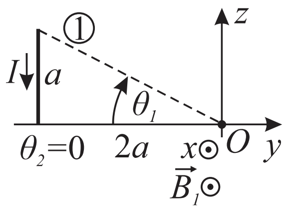
Za položen kolokvijum neophodno je tačno uraditi više od 50% svakog od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

Ispit iz OET2, 27.06.2017.

I-1 **A**



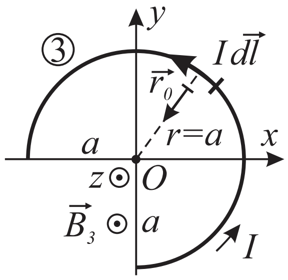
$$\boxed{\vec{B}_2 = 0} \qquad \sin\sphericalangle\left(\vec{dl},\vec{r}_0\right) = 0$$



$\boxed{1}$ $\quad d = 2a, \quad \theta_1 < 0, \quad \theta_2 = 0$

$$B_1 = \frac{\mu_0 I}{4\pi d}\left(\sin\theta_2 - \sin\theta_1\right) = \frac{\mu_0 I}{4\pi 2a}\left[0 - \left(-\frac{a}{\sqrt{a^2 + (2a)^2}}\right)\right] = \frac{\mu_0 I}{4\pi 2a}\frac{\sqrt{5}}{5}$$

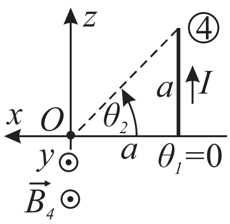
$$\boxed{\vec{B}_1 = \frac{\sqrt{5}\mu_0 I}{40\pi a}\cdot\vec{i}_x}$$



$$\vec{dB}_3 = \frac{\mu_0}{4\pi}\frac{I\vec{dl}\times\vec{r}_0}{r^2} \qquad dB_3 = \frac{\mu_0}{4\pi}\frac{I\,dl}{a^2} \qquad \sphericalangle\left(\vec{dl},\vec{r}_0\right) = \frac{\pi}{2}$$

$$B_3 = \int dB_3 = \frac{\mu_0 I}{4\pi a^2}\int_0^{\frac{3}{4}2a\pi} dl = \frac{\mu_0 I}{4\pi a^2}\frac{3}{2}a\pi = \frac{3\mu_0 I}{8a}$$

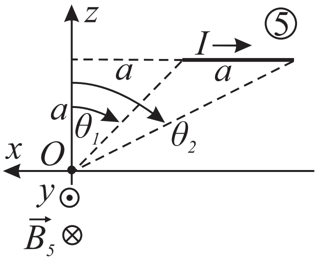
$$\boxed{\vec{B}_3 = \frac{3\mu_0 I}{8a}\cdot\vec{i}_z}$$



$\boxed{4}$ $\quad d = a, \quad \theta_1 = 0, \quad \theta_2 > 0$

$$B_4 = \frac{\mu_0 I}{4\pi d}\left(\sin\theta_2 - \sin\theta_1\right) = \frac{\mu_0 I}{4\pi a}\left[\frac{a}{\sqrt{a^2 + a^2}} - 0\right] = \frac{\mu_0 I}{4\pi a}\frac{\sqrt{2}}{2}$$

$$\boxed{\vec{B}_4 = \frac{\sqrt{2}\mu_0 I}{8\pi a}\cdot\vec{i}_y}$$



$\boxed{5}$ $\quad d = a, \quad \theta_1 > 0, \quad \theta_2 > 0$

$$B_5 = \frac{\mu_0 I}{4\pi d}\left(\sin\theta_2 - \sin\theta_1\right) = \frac{\mu_0 I}{4\pi a}\left(\frac{2a}{\sqrt{(2a)^2 + a^2}} - \frac{a}{\sqrt{a^2 + a^2}}\right) = \frac{\mu_0 I}{4\pi a}\left(\frac{2\sqrt{5}}{5} - \frac{\sqrt{2}}{2}\right)$$

$$\boxed{\vec{B}_5 = \frac{\mu_0 I}{40\pi a}\left(4\sqrt{5} - 5\sqrt{2}\right)\cdot\left(-\vec{i}_y\right)}$$

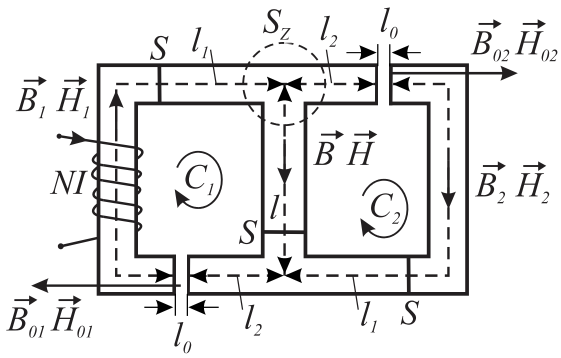
$$\boxed{\vec{B}_O = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 + \vec{B}_4 + \vec{B}_5 = \frac{\sqrt{5}\mu_0 I}{40\pi a}\cdot\vec{i}_x + \left[\frac{\sqrt{2}\mu_0 I}{8\pi a} - \frac{\mu_0 I}{40\pi a}\left(4\sqrt{5} - 5\sqrt{2}\right)\right]\cdot\vec{i}_y + \frac{3\mu_0 I}{8a}\cdot\vec{i}_z}$$

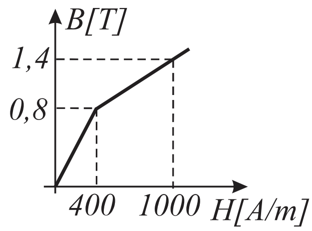
$$\vec{B}_O = 1{,}34\ \mu T\cdot\vec{i}_x + 3{,}12\ \mu T\cdot\vec{i}_y + 28{,}26\ \mu T\cdot\vec{i}_z$$

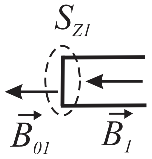
$$\left|\vec{B}_O\right| = \sqrt{(1{,}34\ \mu T)^2 + (3{,}12\ \mu T)^2 + (28{,}26\ \mu T)^2} \qquad \boxed{\left|\vec{B}_O\right| = 28{,}5\ \mu T}$$

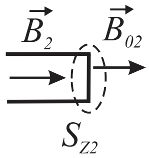
Ispit iz OET2, 27.06.2017.

I-2 **A**

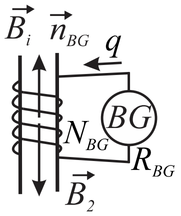








a)



Po Lencovom zakonu, struja indukovana u namotaju kola balističkog galvanometra je takvog smera da se suprotstavlja promeni fluksa.

$$q = -\frac{N_{BG}}{R_{BG}}\left(\Phi_{KR} - \Phi_{POČ}\right)$$

$$\Phi = \int_S \vec{B}\cdot\vec{ds} = -B_2\,S \qquad \sphericalangle\left(\vec{B}_2,\vec{n}_{BG}\right) = \pi$$

$$q = -\frac{N_{BG}}{R_{BG}}\left(B_{POČ}\,S - B_{KR}\,S\right) = \frac{N_{BG}}{R_{BG}}B_{KR}\,S = \frac{N_{BG}}{R_{BG}}B_2\,S \qquad B_{POČ} = 0$$

b)

$$\oint_{S_Z} \vec{B}\cdot\vec{ds} = 0 \qquad\qquad \oint_C \vec{H}\cdot\vec{dl} = NI$$

$$S_Z:\quad -B_1\,S + B_2\,S + B\,S = 0 \qquad (1)$$

$$S_{Z1}:\quad -B_1\,S + B_{01}\,S_0 = 0$$

$$\left(S = S_0\right)\qquad B_1 = B_{01} \qquad (2)$$

$$S_{Z2}:\quad -B_2\,S + B_{02}\,S_0 = 0$$

$$\left(S = S_0\right)\qquad B_2 = B_{02} \qquad (3)$$

$$C_1:\quad H_1\left(l_1 + l_2\right) + H_{01}\,l_0 + H\,l = NI \qquad (4)$$

$$C_2:\quad H_2\left(l_1 + l_2\right) + H_{02}\,l_0 - H\,l = 0 \qquad (5)$$

$$H_0 = \frac{B_0}{\mu_0} \qquad (6)$$

$$B_2 = \frac{q\,R_{BG}}{N_{BG}\,S} = \frac{50\cdot 10^{-6}\cdot 10}{5\cdot 2\cdot 10^{-4}} = 0{,}5\,T$$

Sa krive magnetisanja sledi $H_2 = \dfrac{B_2}{\mu} = \dfrac{0{,}5}{\dfrac{0{,}8}{400}} = 250\ A/m$

Iz (3) sledi $B_{02} = B_2 = 0{,}5\,T \qquad \Rightarrow \qquad H_{02} = \dfrac{B_{02}}{\mu_0} = \dfrac{0{,}5}{4\pi\cdot 10^{-7}} = 398{,}1\,kA/m$

Iz (5) sledi $H = \dfrac{H_2\left(l_1 + l_2\right) + H_{02}\,l_0}{l} = \dfrac{250\cdot\left(24\cdot 10^{-2} + 6\cdot 10^{-2}\right) + 398{,}1\cdot 10^{3}\cdot 0{,}2\cdot 10^{-3}}{10\cdot 10^{-2}} = 1546{,}2\ A/m$

Jednačina za drugi deo krive magnetisanja: $B - B_{K1} = \dfrac{B_{K2} - B_{K1}}{H_{Kc2} - H_{K1}}\left(H - H_{K1}\right)$

$$B - 0{,}8 = \frac{1{,}4 - 0{,}8}{1000 - 400}\left(H - 400\right), \qquad \boxed{B = \frac{0{,}6}{600}H + 0{,}4}$$

Sledi da je $B = \frac{0,6}{600} \cdot 1546,2 + 0,4 = 1,94\,T$

Iz (1) sledi $B_1 = B_2 + B = 2,44\,T$

Sledi da je $H_1 = \frac{600}{0,6}\left(B_1 - 0,4\right) = \frac{600}{0,6}\left(2,44 - 0,4\right) = 2040\,A/m$

Iz (2) sledi $B_{01} = B_1 = 2,44\,T \quad \Rightarrow \quad H_{01} = \frac{B_{01}}{\mu_0} = \frac{2,44}{4\pi \cdot 10^{-7}} = 1942,7\,kA/m$

Iz (4) sledi $NI = H_1\left(l_1 + l_2\right) + H_{01}\,l_0 + H\,l = 2040 \cdot \left(24 \cdot 10^{-2} + 6 \cdot 10^{-2}\right) + 1942,7 \cdot 10^3 \cdot 0,2 \cdot 10^{-3} + 1546,2 \cdot 10 \cdot 10^{-2}$

$\boxed{NI = 1155,16\,Azav}$

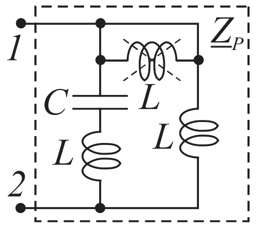
c)

$W_{m2} = \frac{1}{2} B_2\,H_2\,V_2 = \frac{1}{2} B_2\,H_2\left(l_1 + l_2\right) S = \frac{1}{2} \cdot 0,5 \cdot 250 \cdot \left(24 \cdot 10^{-2} + 6 \cdot 10^{-2}\right) \cdot 2 \cdot 10^{-4}$ $\boxed{W_{m2} = 3,75\,mJ}$

Ispit iz OET2, 27.06.2017.

II-1 **A**

a)



$$\underline{Z}_1 = j\omega L + \frac{1}{j\omega C} = \frac{1-\omega^2 LC}{j\omega C}$$

$$\underline{Z}_P = \underline{Z}_1 \parallel j\omega L = \frac{\frac{1-\omega^2 LC}{j\omega C}\, j\omega L}{\frac{1-\omega^2 LC}{j\omega C} + j\omega L} = \frac{\frac{j\omega L\left(1-\omega^2 LC\right)}{j\omega C}}{\frac{1-\omega^2 LC-\omega^2 LC}{j\omega C}} = \frac{j\omega L\left(1-\omega^2 LC\right)}{1-2\omega^2 LC}$$

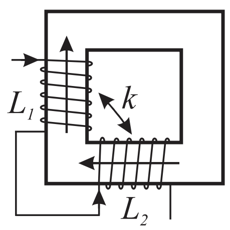
$$\boxed{\underline{Z}_P = j\frac{\omega L\left(1-\omega^2 LC\right)}{1-2\omega^2 LC}}$$

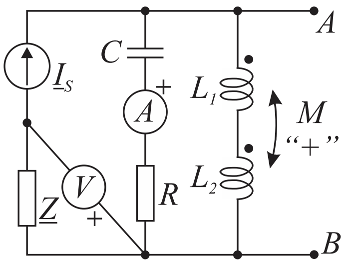
$$\mathrm{Im}\left\{\underline{Z}_P\right\} = 0 \quad \Rightarrow \quad \omega L = 0 \qquad \boxed{\omega_{r1} = 0}$$

$$1-\omega^2 LC = 0 \qquad \boxed{\omega_{r2} = \frac{1}{\sqrt{LC}}}$$

$$\mathrm{Im}\left\{\underline{Y}_P\right\} = 0 \quad \Rightarrow \quad 1-2\omega^2 LC = 0 \qquad \boxed{\omega_{ar1} = \frac{1}{\sqrt{2LC}}} \qquad \boxed{\omega_{ar2} \to \infty}$$

b)

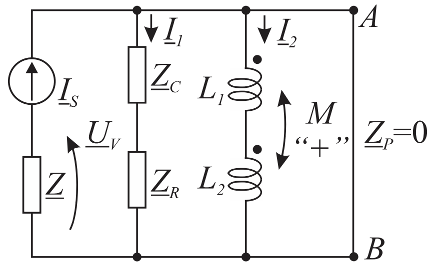




$$M = k\sqrt{L_1 L_2} = k\sqrt{2L\cdot 3L} = 0{,}7\cdot\sqrt{20\,mH\cdot 30\,mH}$$

$$M = 17{,}15\,mH$$

c)



$$\omega_{r2} = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10\cdot 10^{-3}\cdot 40\cdot 10^{-9}}} = 5\cdot 10^4\ \frac{rad}{s} = \omega$$

$$\underline{Z}_P = 0$$

$$\underline{U}_{AB} = 0 \quad \Rightarrow \quad \underline{I}_1 = 0$$

$$\boxed{I_A = \left|\underline{I}_1\right| = 0\ A}$$

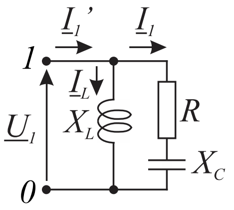
$$\underline{U}_V = \underline{Z}\,\underline{I}_S = (5+j3)\cdot(1-j) = 5+3+j3-j5 = (8-j2)V$$

$$U_V = \left|\underline{U}_V\right| = \sqrt{8^2+(-2)^2} \qquad \boxed{U_V = 8{,}25V}$$

Ispit iz OET2, 27.06.2017.

II-2 **A**

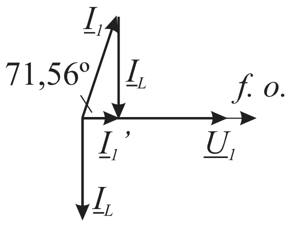
a)



$$\underline{Z} = R - jX_C = (60 - j180)\,\Omega = 189{,}7\,e^{-j71{,}56^\circ}\ \Omega$$

$$\underline{I}_1 = \frac{\underline{U}_1}{\underline{Z}} = \frac{U_1\,e^{j0^\circ}}{Z\,e^{j\varphi}} = \frac{230}{189{,}7\,e^{-j71{,}56^\circ}} = 1{,}21\,e^{j71{,}56^\circ}\ A$$

$$\underline{I}_L = \frac{\underline{U}_1\,e^{j0}}{jX_L} = \frac{U_1\,e^{j0}}{X_L\,e^{j\frac{\pi}{2}}} = \frac{U_1}{X_L}\,e^{-j\frac{\pi}{2}}\ A$$



$$I_L = I_1\,\sin 71{,}56^\circ$$

$$\frac{U_1}{X_L} = \frac{U_1}{Z}\sin 71{,}56^\circ \quad\Rightarrow\quad \frac{1}{X_L} = \frac{\sin 71{,}56^\circ}{Z} \quad\Rightarrow\quad X_L = \frac{189{,}7}{\sin 71{,}56^\circ}$$

$$\boxed{X_L = 199{,}97\,\Omega}$$

$$I_1' = I_1\,\cos 71{,}56^\circ = 1{,}21\cdot\cos 71{,}56^\circ = 0{,}38\ A \qquad \underline{I}_1' = 0{,}38\,e^{j0^\circ}\ A$$

I KZ: $\underline{I}_1' = \underline{I}_L + \underline{I}_1$

$$\frac{U_1}{R_e} = \frac{U_1}{Z}\cos 71{,}56^\circ \quad\Rightarrow\quad \frac{1}{R_e} = \frac{\cos 71{,}56^\circ}{Z} \quad\Rightarrow\quad R_e = \frac{Z}{\cos 71{,}56^\circ} = \frac{189{,}7}{\cos 71{,}56^\circ}$$

$$R_e = 599{,}73\,\Omega$$

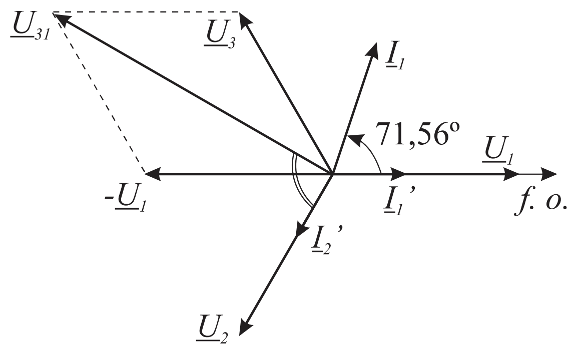
b)

$$\boxed{I_A^{(b)} = |\underline{I}_1'| = 0{,}38\ A}$$

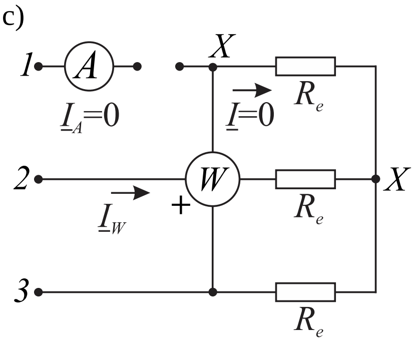
$$P_W^{(b)} = \mathrm{Re}\left\{\underline{U}_{31}\,\underline{I}_2'^{\,*}\right\} = U_{31}\,I_2'\cos\sphericalangle\left(\underline{U}_{31}, \underline{I}_2'\right)$$

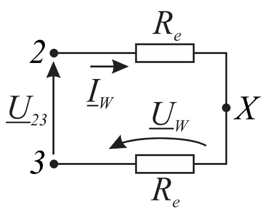
$$P_W^{(b)} = \sqrt{3}\,U_1\,I_1'\cos\left(30^\circ + 60^\circ\right) = \sqrt{3}\cdot 230\cdot 0{,}38\cdot\cos 90^\circ$$

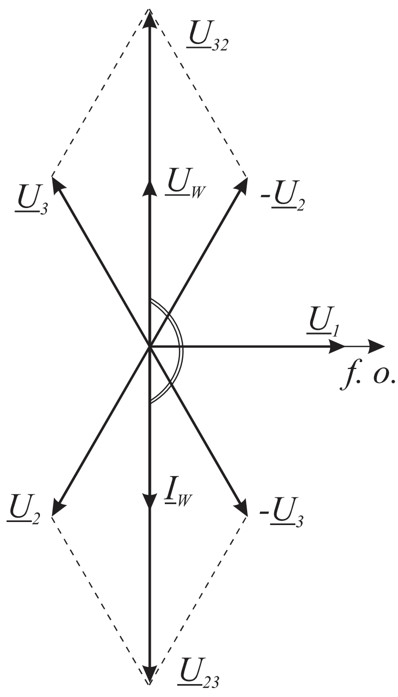
$$\boxed{P_W^{(b)} = 0\,W}$$



c)







$$\boxed{I_A^{(c)} = 0\ A}$$

$$\underline{U}_W = \underline{U}_{3X} = \frac{1}{2}\underline{U}_{32} = \frac{1}{2}\sqrt{3}\cdot 230\,e^{j90^\circ}\ V$$

$$\underline{I}_W = \frac{\underline{U}_{23}}{2R_e} = \frac{\sqrt{3}\cdot 230\,e^{-j90^\circ}}{2\cdot 599{,}73} = 0{,}33\,e^{-j90^\circ}\ A$$

$$P_W^{(c)} = \mathrm{Re}\left\{\underline{U}_W\,\underline{I}_W^{\,*}\right\} = U_W\,I_W\cos\sphericalangle\left(\underline{U}_W, \underline{I}_W\right)$$

$$P_W^{(c)} = \frac{1}{2}\sqrt{3}\cdot 230\cdot 0{,}33\cdot\cos 180^\circ$$

$$\boxed{P_W^{(c)} = -65{,}73\,W}$$

STUDIJSKI PROGRAM ENERGETIKA, ELEKTRONIKA I TELEKOMUNIKACIJE

K TET

Katedra za teorijsku elektrotehniku

www.ktet.ftn.uns.ac.rs

Osnovi elektrotehnike 2 (I kolokvijum)

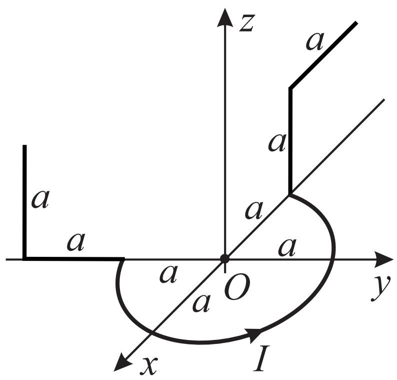
K1 - B

27.06.2017.

ZADACI

Zadatak 1. Žičani provodnik konačne dužine, sa vremenski konstantnom strujom jačine I, savijen je kao što je prikazano na slici 1. Provodnik se sastoji od četiri pravolinijska i jednog lučnog segmenta, kao što je prikazano na slici. Dva pravolinijska segmetna leže u y-z ravni, dok preostala dva u x-z ravni, Dekartovog pravouglog koordinatnog sistema. Lučni segment je u obliku 3/4 kruga, poluprečnika a, i leži u x-y ravni. Odrediti intenzitet vektora magnetske indukcije u koordinatnom početku. Sredina je vazduh.

Brojne vrednosti: $I = 3\ A$, $a = 5\ cm$.



Slika 1.

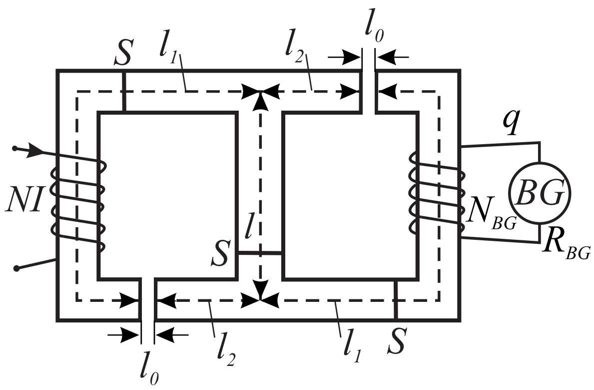
Zadatak 2. Tanko magnetsko kolo sa slike 2a je načinjeno od materijala, čija je idealizovana kriva prvobitnog magnetisanja data na slici 2b.

a) Odrediti smer protekle količine naelektrisanja, q, prilikom uspostavljanja magnetskog polja u jezgru. Pre uključivanja struje, jezgro je bilo nenamagnetisano.

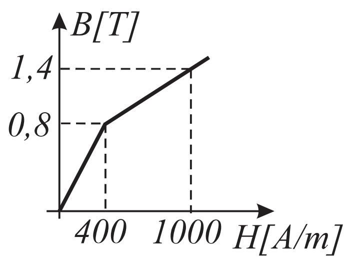
b) Izračunati broj amperzavojaka, NI, kako bi, prilikom uspostavljanja magnetskog polja u jezgru, kroz kroz kolo balističkog galvanometra protekla količina naelektrisanja $q = 50\ \mu C$, u smeru određenom pod a). Rasipanja magnetskog fluksa zanemariti.

c) Odrediti energiju utrošenu na uspostavljanje magnetskog polja u grani sa balističkim galvanometrom.

Brojni podaci: $l_0 = 0{,}2\ mm$, $l_1 = 24\ cm$, $l_2 = 6\ cm$, $l = 10\ cm$, $S = 2\ cm^2$, $N_{BG} = 5$ zavojaka, $R_{BG} = 10\ \Omega$.



Slika 2a.



Slika 2b.

PRAVILA POLAGANJA

Za položen kolokvijum neophodno je tačno uraditi više od 50% svakog od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

STUDIJSKI PROGRAM ENERGETIKA, ELEKTRONIKA I TELEKOMUNIKACIJE

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Osnovi elektrotehnike 2 (I kolokvijum)

K2 - B

27.06.2017.

ZADACI

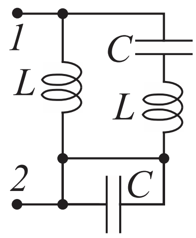
Zadatak 1. Prijemnik sa slike 1a treba da se priključi na ulazni deo kola sa slike 1b, između tačaka *A* i *B*.

a) Odrediti sve rezonantne i antirezonantne kružne učestanosti prijemnika prikazanog na slici 1a.

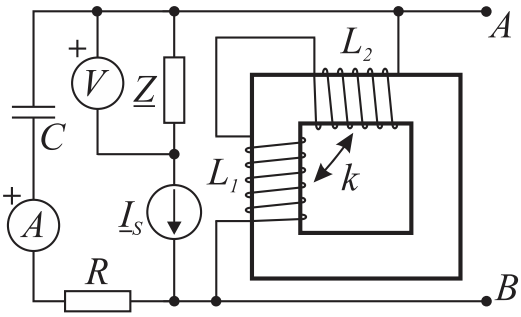
b) Nacrtati električnu šemu ulaznog dela kola i u njoj označiti tip magnetske sprege koji postoji između namotaja induktivnosti L_1 i L_2 (koristiti simbol tačkica).

c) Odrediti pokazivanja idealnih mernih instrumenata, kada je prijemnik priključen na ulazni deo kola, pri čemu je ugaona učestanost struje strujnog generatora $\underline{I}_S$ jednaka $5 \cdot 10^4$ *rad/s*.

Brojni podaci: $L = 10\ mH$, $C = 40\ nF$, $R = 30\ \Omega$, $\underline{Z} = (5 + j3)\ \Omega$, $\underline{I}_S = (1 - j)\ \Omega$, $L_1 = 2L$, $L_2 = 3L$, $k = 0{,}7$.



Slika 1a.



Slika 1b.

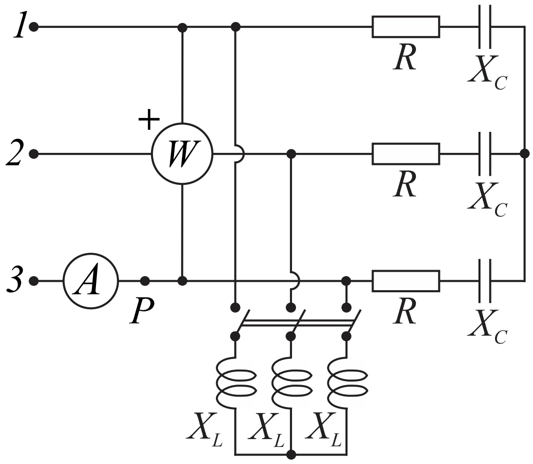
Zadatak 2. Na slici 2 je prikazan simetričan trofazni prijemnik, sastavljen od otpornika otpornosti $R = 30\ \Omega$ i kondenzatora reaktanse $X_C = 90\ \Omega$. Prijemnik je priključen na mrežu faznog napona $\underline{U}_1 = 230\ V$.

a) Odrediti reaktanse kalemova, koje je potrebno povezati, kako bi se faktor snage prijemnika popravio na jedinicu.

b) Odrediti pokazivanja idealnih mernih instrumenata posle priključivanja kalemova.

c) Odrediti pokazivanja idealnih mernih instrumenata, dok su kalemovi priključeni, ali kada se desi prekid provodnika treće faze, u tački *P*.

d) Na istom fazorskom dijagramu prikazati fazore svih veličina od kojih zavise pokazivanja instrumenata.



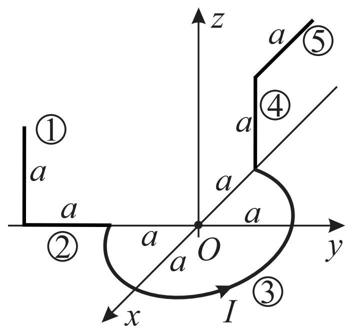
Slika 2.

PRAVILA POLAGANJA

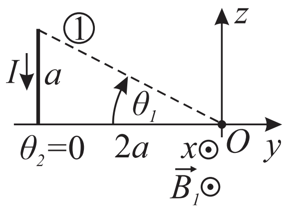
Za položen kolokvijum neophodno je tačno uraditi više od 50% svakog od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

Ispit iz OET2, 27.06.2017.

I-1 **B**



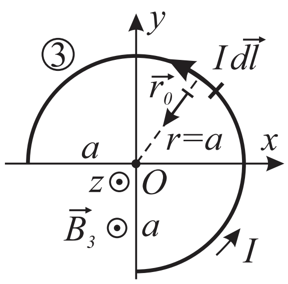
$$\boxed{\vec{B}_2 = 0} \qquad \sin\sphericalangle\left(\vec{dl},\vec{r}_0\right) = 0$$



$\boxed{1}$ $\qquad d = 2a, \qquad \theta_1 < 0, \qquad \theta_2 = 0$

$$B_1 = \frac{\mu_0 I}{4\pi d}\left(\sin\theta_2 - \sin\theta_1\right) = \frac{\mu_0 I}{4\pi 2a}\left[0 - \left(-\frac{a}{\sqrt{a^2 + (2a)^2}}\right)\right] = \frac{\mu_0 I}{4\pi 2a}\frac{\sqrt{5}}{5}$$

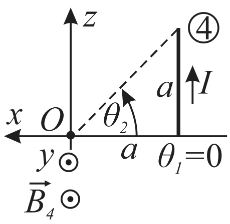
$$\boxed{\vec{B}_1 = \frac{\sqrt{5}\mu_0 I}{40\pi a}\cdot\vec{i}_x}$$



$$\vec{dB}_3 = \frac{\mu_0}{4\pi}\frac{I\vec{dl}\times\vec{r}_0}{r^2} \qquad dB_3 = \frac{\mu_0}{4\pi}\frac{I\,dl}{a^2} \qquad \sphericalangle\left(\vec{dl},\vec{r}_0\right) = \frac{\pi}{2}$$

$$B_3 = \int dB_3 = \frac{\mu_0 I}{4\pi a^2}\int_0^{\frac{3}{4}2a\pi} dl = \frac{\mu_0 I}{4\pi a^2}\frac{3}{2}a\pi = \frac{3\mu_0 I}{8a}$$

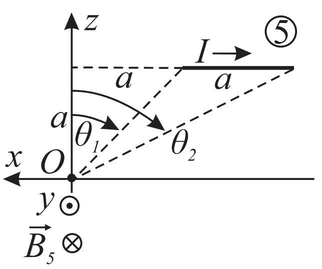
$$\boxed{\vec{B}_3 = \frac{3\mu_0 I}{8a}\cdot\vec{i}_z}$$



$\boxed{4}$ $\qquad d = a, \qquad \theta_1 = 0, \qquad \theta_2 > 0$

$$B_4 = \frac{\mu_0 I}{4\pi d}\left(\sin\theta_2 - \sin\theta_1\right) = \frac{\mu_0 I}{4\pi a}\left[\frac{a}{\sqrt{a^2 + a^2}} - 0\right] = \frac{\mu_0 I}{4\pi a}\frac{\sqrt{2}}{2}$$

$$\boxed{\vec{B}_4 = \frac{\sqrt{2}\mu_0 I}{8\pi a}\cdot\vec{i}_y}$$



$\boxed{5}$ $\qquad d = a, \qquad \theta_1 > 0, \qquad \theta_2 > 0$

$$B_5 = \frac{\mu_0 I}{4\pi d}\left(\sin\theta_2 - \sin\theta_1\right) = \frac{\mu_0 I}{4\pi a}\left(\frac{2a}{\sqrt{(2a)^2 + a^2}} - \frac{a}{\sqrt{a^2 + a^2}}\right) = \frac{\mu_0 I}{4\pi a}\left(\frac{2\sqrt{5}}{5} - \frac{\sqrt{2}}{2}\right)$$

$$\boxed{\vec{B}_5 = \frac{\mu_0 I}{40\pi a}\left(4\sqrt{5} - 5\sqrt{2}\right)\cdot\left(-\vec{i}_y\right)}$$

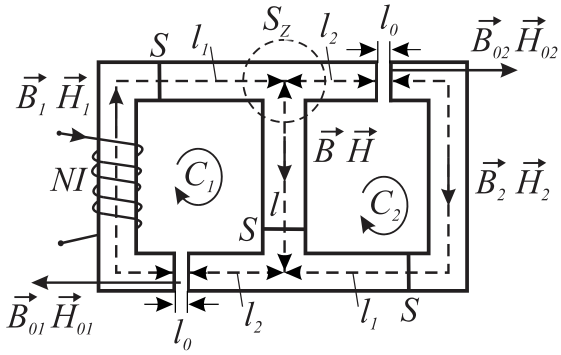
$$\boxed{\vec{B}_O = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 + \vec{B}_4 + \vec{B}_5 = \frac{\sqrt{5}\mu_0 I}{40\pi a}\cdot\vec{i}_x + \left[\frac{\sqrt{2}\mu_0 I}{8\pi a} - \frac{\mu_0 I}{40\pi a}\left(4\sqrt{5} - 5\sqrt{2}\right)\right]\cdot\vec{i}_y + \frac{3\mu_0 I}{8a}\cdot\vec{i}_z}$$

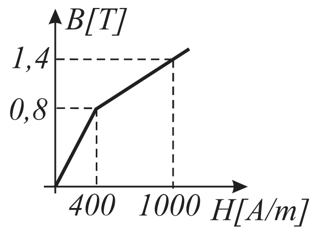
$$\vec{B}_O = 1{,}34\ \mu T\cdot\vec{i}_x + 3{,}12\ \mu T\cdot\vec{i}_y + 28{,}26\ \mu T\cdot\vec{i}_z$$

$$\left|\vec{B}_O\right| = \sqrt{(1{,}34\ \mu T)^2 + (3{,}12\ \mu T)^2 + (28{,}26\ \mu T)^2} \qquad \boxed{\left|\vec{B}_O\right| = 28{,}5\ \mu T}$$

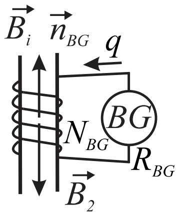
Ispit iz OET2, 27.06.2017.

I-2 **B**





a)



Po Lencovom zakonu, struja indukovana u namotaju kola balističkog galvanometra je takvog smera da se suprotstavlja promeni fluksa.

$$q=-\frac{N_{BG}}{R_{BG}}\left(\Phi_{KR}-\Phi_{POČ}\right)$$

$$\Phi=\int_S \vec{B}\cdot\vec{ds}=-B_2\,S \qquad \sphericalangle\left(\vec{B}_2,\vec{n}_{BG}\right)=\pi$$

$$q=-\frac{N_{BG}}{R_{BG}}\left(B_{POČ}\,S-B_{KR}\,S\right)=\frac{N_{BG}}{R_{BG}}B_{KR}\,S=\frac{N_{BG}}{R_{BG}}B_2\,S \qquad B_{POČ}=0$$

b)

$$\oint_{S_Z}\vec{B}\cdot\vec{ds}=0$$

$S_Z:\quad -B_1 S+B_2 S+B\,S=0 \qquad (1)$

$S_{Z1}:\quad -B_1 S+B_{01} S_0=0$

$\left(S=S_0\right)\qquad B_1=B_{01} \qquad (2)$

$S_{Z2}:\quad -B_2 S+B_{02} S_0=0$

$\left(S=S_0\right)\qquad B_2=B_{02} \qquad (3)$

$$\oint_C \vec{H}\cdot\vec{dl}=NI$$

$C_1:\quad H_1\left(l_1+l_2\right)+H_{01}\,l_0+H\,l=NI \qquad (4)$

$C_2:\quad H_2\left(l_1+l_2\right)+H_{02}\,l_0-H\,l=0 \qquad (5)$

$H_0=\dfrac{B_0}{\mu_0} \qquad (6)$

$$B_2=\frac{q\,R_{BG}}{N_{BG}\,S}=\frac{50\cdot10^{-6}\cdot10}{5\cdot2\cdot10^{-4}}=0,5\,T$$

Sa krive magnetisanja sledi $H_2=\dfrac{B_2}{\mu}=\dfrac{0,5}{\frac{0,8}{400}}=250\,A/m$

Iz (3) sledi $B_{02}=B_2=0,5\,T \quad\Rightarrow\quad H_{02}=\dfrac{B_{02}}{\mu_0}=\dfrac{0,5}{4\pi\cdot10^{-7}}=398,1\,kA/m$

Iz (5) sledi $H=\dfrac{H_2\left(l_1+l_2\right)+H_{02}\,l_0}{l}=\dfrac{250\cdot\left(24\cdot10^{-2}+6\cdot10^{-2}\right)+398,1\cdot10^3\cdot0,2\cdot10^{-3}}{10\cdot10^{-2}}=1546,2\,A/m$

Jednačina za drugi deo krive magnetisanja: $B-B_{K1}=\dfrac{B_{K2}-B_{K1}}{H_{Kc2}-H_{K1}}\left(H-H_{K1}\right)$

$$B-0,8=\frac{1,4-0,8}{1000-400}\left(H-400\right), \qquad \boxed{B=\frac{0,6}{600}H+0,4}$$

Sledi da je $B = \frac{0,6}{600} \cdot 1546,2 + 0,4 = 1,94\,T$

Iz (1) sledi $B_1 = B_2 + B = 2,44\,T$

Sledi da je $H_1 = \frac{600}{0,6}\left(B_1 - 0,4\right) = \frac{600}{0,6}\left(2,44 - 0,4\right) = 2040\,A/m$

Iz (2) sledi $B_{01} = B_1 = 2,44\,T$ $\Rightarrow$ $H_{01} = \frac{B_{01}}{\mu_0} = \frac{2,44}{4\pi \cdot 10^{-7}} = 1942,7\,kA/m$

Iz (4) sledi $NI = H_1\left(l_1 + l_2\right) + H_{01}\,l_0 + H\,l = 2040 \cdot \left(24 \cdot 10^{-2} + 6 \cdot 10^{-2}\right) + 1942,7 \cdot 10^3 \cdot 0,2 \cdot 10^{-3} + 1546,2 \cdot 10 \cdot 10^{-2}$

$\boxed{NI = 1155,16\,Azav}$

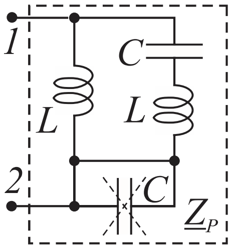
c)

$W_{m2} = \frac{1}{2} B_2\,H_2\,V_2 = \frac{1}{2} B_2\,H_2\left(l_1 + l_2\right)S = \frac{1}{2} \cdot 0,5 \cdot 250 \cdot \left(24 \cdot 10^{-2} + 6 \cdot 10^{-2}\right) \cdot 2 \cdot 10^{-4}$ $\boxed{W_{m2} = 3,75\,mJ}$

Ispit iz OET2, 27.06.2017.

II-1 **B**

a)



$$\underline{Z}_1 = j\omega L + \frac{1}{j\omega C} = \frac{1-\omega^2 LC}{j\omega C}$$

$$\underline{Z}_P = \underline{Z}_1 \parallel j\omega L = \frac{\frac{1-\omega^2 LC}{j\omega C}\, j\omega L}{\frac{1-\omega^2 LC}{j\omega C} + j\omega L} = \frac{\frac{j\omega L\left(1-\omega^2 LC\right)}{j\omega C}}{\frac{1-\omega^2 LC-\omega^2 LC}{j\omega C}} = \frac{j\omega L\left(1-\omega^2 LC\right)}{1-2\omega^2 LC}$$

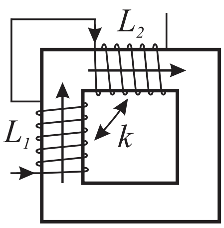
$$\boxed{\underline{Z}_P = j\frac{\omega L\left(1-\omega^2 LC\right)}{1-2\omega^2 LC}}$$

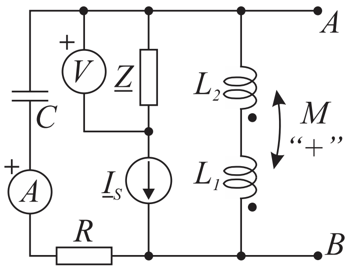
$$\mathrm{Im}\left\{\underline{Z}_P\right\} = 0 \quad \Rightarrow \quad \omega L = 0 \qquad \boxed{\omega_{r1} = 0}$$

$$1-\omega^2 LC = 0 \qquad \boxed{\omega_{r2} = \frac{1}{\sqrt{LC}}}$$

$$\mathrm{Im}\left\{\underline{Y}_P\right\} = 0 \quad \Rightarrow \quad 1-2\omega^2 LC = 0 \qquad \boxed{\omega_{ar1} = \frac{1}{\sqrt{2LC}}} \qquad \boxed{\omega_{ar2} \to \infty}$$

b)

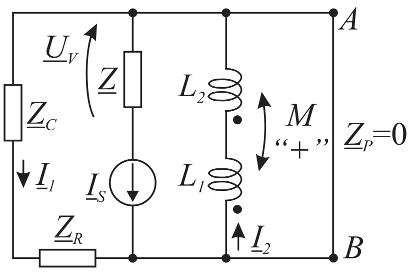




$$M = k\sqrt{L_1 L_2} = k\sqrt{2L\cdot 3L} = 0{,}7\cdot\sqrt{20\,mH\cdot 30\,mH}$$

$$M = 17{,}15\,mH$$

c)



$$\omega_{r2} = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10\cdot 10^{-3}\cdot 40\cdot 10^{-9}}} = 5\cdot 10^4\ \frac{rad}{s} = \omega$$

$$\underline{Z}_P = 0$$

$$\underline{U}_{AB} = 0 \quad \Rightarrow \quad \underline{I}_1 = 0$$

$$\boxed{I_A = \left|\underline{I}_1\right| = 0\ A}$$

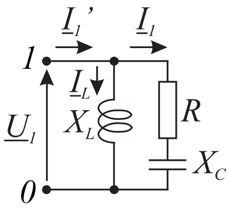
$$\underline{U}_V = \underline{Z}\,\underline{I}_S = \left(5+j3\right)\cdot\left(1-j\right) = 5+3+j3-j5 = \left(8-j2\right)V$$

$$U_V = \left|\underline{U}_V\right| = \sqrt{8^2+\left(-2\right)^2} \qquad \boxed{U_V = 8{,}25\,V}$$

Ispit iz OET2, 27.06.2017.

II-2 **B**

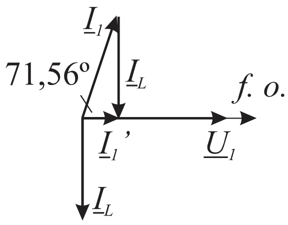
a)



$$\underline{Z} = R - jX_C = (30 - j90)\,\Omega = 94{,}9\, e^{-j71{,}56^\circ}\ \Omega$$

$$\underline{I}_1 = \frac{\underline{U}_1}{\underline{Z}} = \frac{U_1\, e^{j0^\circ}}{Z\, e^{j\varphi}} = \frac{230}{94{,}9\, e^{-j71{,}56^\circ}\ \Omega} = 2{,}42\, e^{j71{,}56^\circ}\ A$$

$$\underline{I}_L = \frac{\underline{U}_1\, e^{j0}}{jX_L} = \frac{U_1\, e^{j0}}{X_L\, e^{j\frac{\pi}{2}}} = \frac{U_1}{X_L}\, e^{-j\frac{\pi}{2}}\ A$$



$$I_L = I_1 \sin 71{,}56^\circ$$

$$\frac{U_1}{X_L} = \frac{U_1}{Z}\sin 71{,}56^\circ \quad\Rightarrow\quad \frac{1}{X_L} = \frac{\sin 71{,}56^\circ}{Z} \quad\Rightarrow\quad X_L = \frac{94{,}9}{\sin 71{,}56^\circ}$$

$$\boxed{X_L = 100{,}04\,\Omega}$$

$$I_1' = I_1 \cos 71{,}56^\circ = 2{,}42\cdot\cos 71{,}56^\circ = 0{,}77\ A \qquad \underline{I}_1' = 0{,}77\, e^{j0^\circ}\ A$$

I KZ: $\underline{I}_1' = \underline{I}_L + \underline{I}_1$

$$\frac{U_1}{R_e} = \frac{U_1}{Z}\cos 71{,}56^\circ \quad\Rightarrow\quad \frac{1}{R_e} = \frac{\cos 71{,}56^\circ}{Z} \quad\Rightarrow\quad R_e = \frac{Z}{\cos 71{,}56^\circ} = \frac{94{,}9}{\cos 71{,}56^\circ}$$

$$R_e = 300{,}02\,\Omega$$

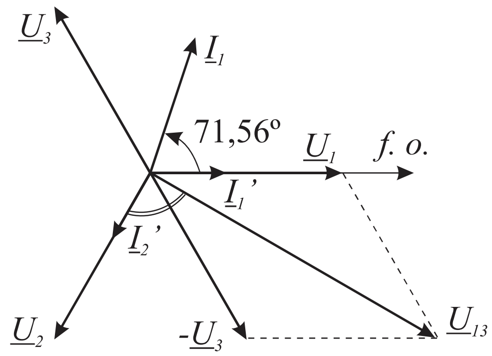
b)

$$\boxed{I_A^{(b)} = \left|\underline{I}_3'\right| = \left|\underline{I}_1'\right| = 0{,}77\ A}$$

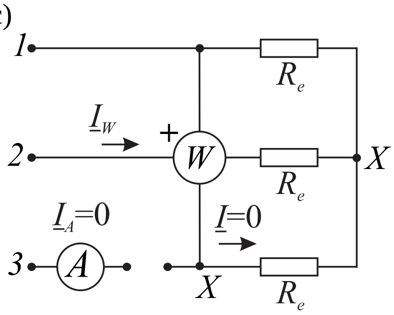
$$P_W^{(b)} = \mathrm{Re}\left\{\underline{U}_{13}\ \underline{I}_2'^{\,*}\right\} = U_{13}\, I_2' \cos\sphericalangle\left(\underline{U}_{13}, \underline{I}_2'\right)$$

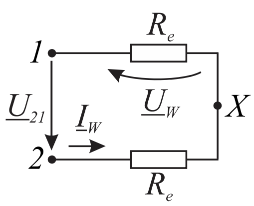
$$P_W^{(b)} = \sqrt{3}\, U_1\, I_1' \cos\left(120^\circ - 30^\circ\right) = \sqrt{3}\cdot 230\cdot 0{,}77\cdot\cos 90^\circ$$

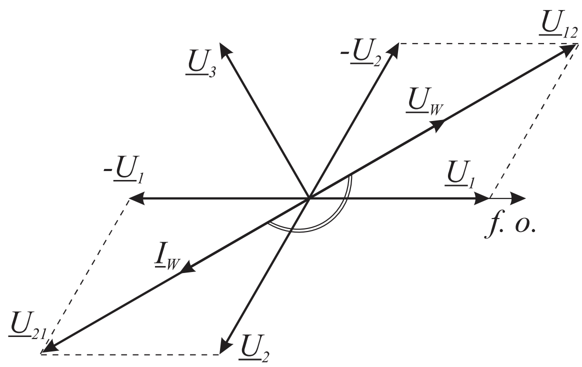
$$\boxed{P_W^{(b)} = 0\,W}$$



c)







$$\boxed{I_A^{(c)} = 0\ A}$$

$$\underline{U}_W = \underline{U}_{1X} = \frac{1}{2}\underline{U}_{12} = \frac{1}{2}\sqrt{3}\cdot 230\, e^{j30^\circ}\ V$$

$$\underline{I}_W = \frac{\underline{U}_{21}}{2R_e} = \frac{\sqrt{3}\cdot 230\, e^{-j150^\circ}}{2\cdot 300{,}02} = 0{,}66\, e^{-j150^\circ}\ A$$

$$P_W^{(c)} = \mathrm{Re}\left\{\underline{U}_W\ \underline{I}_W^{\,*}\right\} = U_W\, I_W \cos\sphericalangle\left(\underline{U}_W, \underline{I}_W\right)$$

$$P_W^{(c)} = \frac{1}{2}\sqrt{3}\cdot 230\cdot 0{,}66\cdot\cos 180^\circ$$

$$\boxed{P_W^{(c)} = -131{,}46\,W}$$