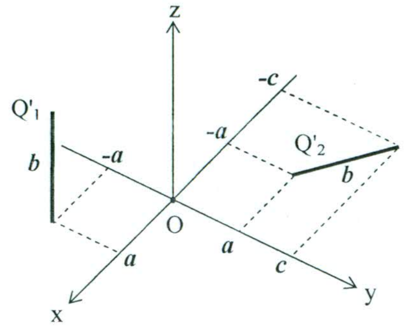
PRVI DEO ISPITA IZ OSNOVA ELEKTROTEHNIKE 1
za studijske programe: Energetika, elektronika i telekomunikacije i Merenje i regulacija
Novi Sad, 28. januar 2015.
(I kolokvijum)

I-1. Dva tanaka, prava štapa, dužine b=*7cm*, naelektrisana ravnomerno podužnim naelektrisanjima Q'$_1$ i Q'$_2$, postavljena su u zadati koordinatni sistem kao na slici 1. Prvi štap je upravan na x-y ravan, a jedan njegov kraj leži u toj ravni. Drugi štap leži u x-y ravani. Poznato je podužno naelektrisanje prvog štapa Q'$_1$= 2,5nC/m i koordinata a=4cm. Sredina je vazduh.

a) Odrediti podužno naelektrisanje drugog štapa, Q'$_2$, tako da rezultantni vektor jačine električnog polja u tački **O** (***koordinatni početak***) ima samo z komponentu.

b) Izračunati brojnu vrednost intenziteta z komponente vektora jačine električnog polja u tački O.



Slika 1.

Permitivnost vakuuma je $\varepsilon_0 = 8{,}85 \cdot 10^{-12} F/m$.

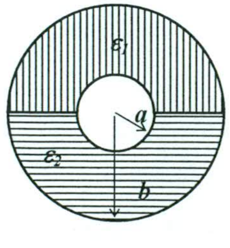
I-2. <u>**Sferni kondenzator**</u>, čiji je presek prikazan na slici 2, ima dva sloja dielektrika, relativnih permitivnosti ε_{r1}=2 i ε_{r1}=9. Elektrode kondenzatora su naelektrisane naelektrisanjem Q i –Q. Poluprečnici elektroda su a=1cm i b=2,5cm. Poznata je maksimalna jačina električnog polja u kondenzatoru E_{max}=350kV/m.

a) Razmotriti granične uslove i odrediti kako se u zavisnosti rastojanja tačke od centra kondenzatora menjaju intenziteti vektora električnog pomeraja i vektora jačine električnog polja.

b) Odrediti naelektrisanje elektroda kondenzatora.

c) Izračunati napon između elektroda kondenzatora.

d) Izračunati gustine vezanih naelektrisanja na površima dielektrika koje se oslanjaju na unutrašnju elektrodu.



Slika 2.

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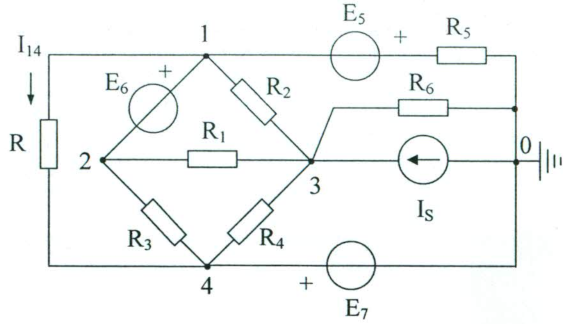
II-1. U mreži sa slike 3 poznate su sledeće brojne vrednosti: E_5=12V, E_6=E_7=6V, I_S=120mA, R_1=R_2=100Ω, R_3=R_5=300 Ω, R_4=R_6=200Ω. Odrediti:

(a) vrednost otpornosti otpornika R, tako da jačina struje kroz njegove priključke ima vrednost I_{14}=20mA;

(b) snage strujnog generatora i naponskog generatora ems E_7.

Pri rešavanju zadatka obavezno primeniti teoremu kompenzacije i metodu po kojoj treba postaviti sistem jednačina sa minimalnim brojem nepoznatih.



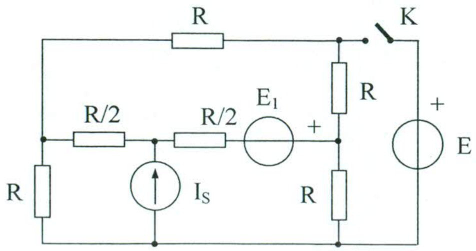
Slika 3.

II-2. Zatvaranjem prekidača K u mrežu, prikazanu na slici 4, se uključuje idealni naponski generator elektromotorne sile E=10V. Jačina struje kroz priključke ovog generatora ne sme da bude veća od I_{max}=25mA. Zadate su i sledeće brojne vrednosti: E_1=12V, I_S=100mA, R=100Ω.

(a) Proveriti da li će, nakon zatvaranja prekidača, jačina struje kroz generator preći graničnu vrednost.

(b) Po potrebi, dodati redno vezani zaštitni otpornik i odrediti njegovu otpornost. Izračunati, takođe, graničnu vrednost dozvoljene snage zaštitnog otpornika.

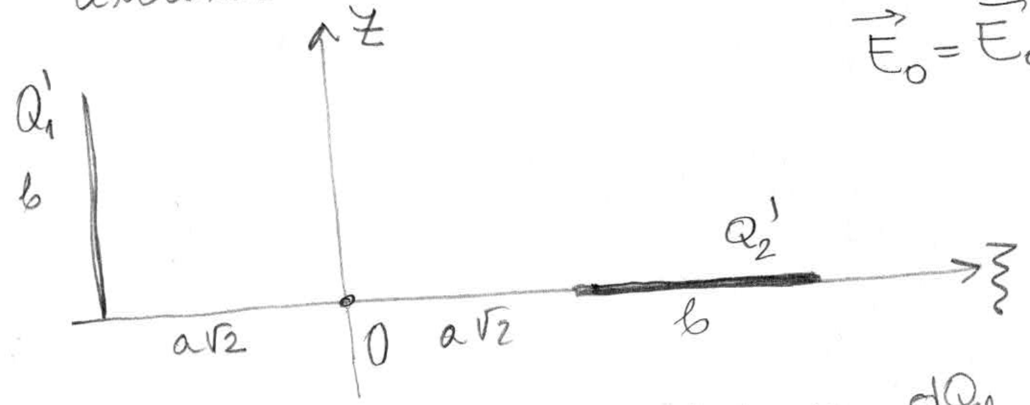
Pri rešavanju zadatka obavezno primeniti Tevenenovu teoremu i metodu po kojoj treba postaviti sistem jednačina sa minimalnim brojem nepoznatih.



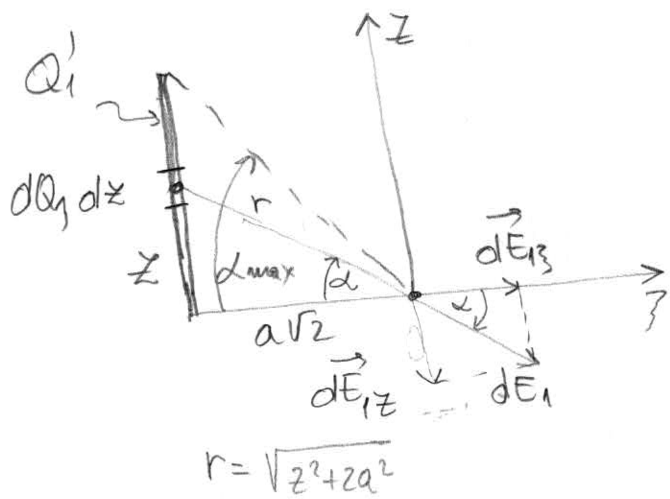
Slika 4.

I-1. Испит 28. јануар 2015.

После увођења помоћне координатне система имамо



$$\vec{E}_0 = \vec{E}_{01} + \vec{E}_{02}$$



$$dE_1 = \frac{dQ_1}{4\pi\varepsilon_0 r^2} \qquad dE_{1\xi} = dE_1 \cos\alpha \qquad dE_{1z} = dE_1 \sin\alpha$$

$$E_{1\xi} = \int_{z=0}^{z=b} \frac{Q_1' dz \cos\alpha}{4\pi\varepsilon_0 r^2} = \frac{Q_1'}{4\pi\varepsilon_0 a\sqrt{2}} \int_{\alpha=0}^{\alpha=\alpha_{max}} \cos\alpha \, d\alpha$$

$$dz\cos\alpha = r\,d\alpha \qquad r = \frac{a\sqrt{2}}{\cos\alpha}$$

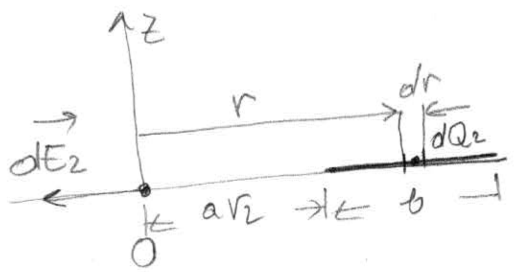
$$E_{1\xi} = \frac{Q_1'}{4\pi\varepsilon_0 a\sqrt{2}} \sin\alpha_{max} = \frac{Q_1'}{4\pi\varepsilon_0 a\sqrt{2}} \frac{b}{\sqrt{b^2+2a^2}}$$

5

$$r = \sqrt{z^2+2a^2}$$

$$E_{1z} = \int_{z=0}^{z=b} \frac{Q_1' dz \sin\alpha}{4\pi\varepsilon_0 r^2} = \frac{Q_1'}{4\pi\varepsilon_0} \int_{z=0}^{z=b} \frac{z\,dz}{(z^2+2a^2)^{3/2}}$$

$$\sin\alpha = \frac{z}{r}$$



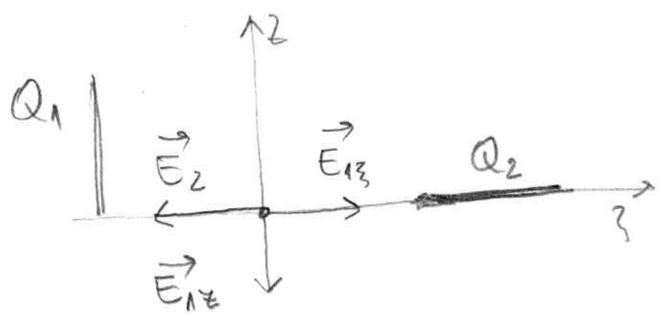
$$E_{1z} = \frac{Q_1'}{4\pi\varepsilon_0}\left(-\frac{1}{\sqrt{z^2+2a^2}}\right)\Bigg|_{z=0}^{z=b} = \frac{Q_1'}{4\pi\varepsilon_0}\left(\frac{1}{a\sqrt{2}} - \frac{1}{\sqrt{b^2+2a^2}}\right)$$

5

$$dE_2 = \frac{dQ_2}{4\pi\varepsilon_0 r^2} = \frac{Q_2' \cdot dr}{4\pi\varepsilon_0 r^2}$$

$$E_2 = \int_{duž\ st.2} dE_2 = \frac{Q_2'}{4\pi\varepsilon_0} \int_{r=a\sqrt{2}}^{r=a\sqrt{2}+b} \frac{dr}{r^2} = \frac{Q_2'}{4\pi\varepsilon_0}\left(\frac{1}{a\sqrt{2}} - \frac{1}{a\sqrt{2}+b}\right)$$

5



За да $\vec{E}_0 = \vec{E}_{0z}$ мора бити:

$$E_{1\xi} = E_2$$

$$\frac{Q_1'}{4\pi\varepsilon_0 a\sqrt{2}} \frac{b}{\sqrt{b^2+2a^2}} = \frac{Q_2'}{4\pi\varepsilon_0}\left(\frac{1}{a\sqrt{2}} - \frac{1}{a\sqrt{2}+b}\right)$$

$$\frac{Q_1'}{a\sqrt{2}} \frac{\cancel{b}}{\sqrt{b^2+2a^2}} = Q_2' \frac{\cancel{b}}{a\sqrt{2}\,(a\sqrt{2}+b)}$$

$$Q_2' = \frac{a\sqrt{2}+b}{\sqrt{2a^2+b^2}} Q_1'$$

$$Q_2' = \frac{4\sqrt{2}+7}{\sqrt{32+49}} \cdot 2{,}5 \ nC/m$$

$$\boxed{Q_2' \cong 3{,}5 \ nC/m}$$

7 ~~5~~

b)

$$E_0 = E_{1z} = \frac{Q_1'}{4\pi\varepsilon_0}\Big(\underbrace{\frac{1}{a\sqrt{2}}}_{17{,}68} - \frac{1}{\sqrt{2a^2+b^2}}\Big) \quad -11{,}11$$

$$E_0 = 2{,}5 \cdot 10^{-9} \cdot 9 \cdot 10^{9} \cdot 6{,}57 = 147{,}825 \ V/m$$

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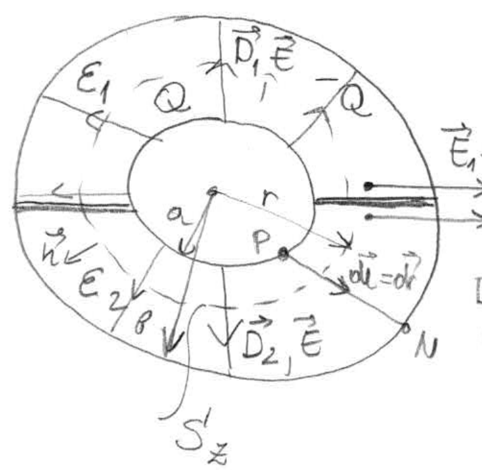
$$\vec{E}_0 = E_0 \cdot (-\vec{u}_z)$$

3

I-2. Испит 28.01.2015.

a)



(2)

$$\left.\begin{array}{l}\vec{E}_1 = \vec{E}_{t1} \\ \vec{E}_2 = \vec{E}_{t2}\end{array}\right\} \Rightarrow \begin{array}{l} E_{t1} = E_{t2} \\ E_1(r) = E_2(r) = E(r)\end{array}$$

$$\left.\begin{array}{l} D_{n1} = 0 \\ D_{n2} = 0 \end{array}\right\} D_{n1} = D_{n2}$$

$$D_1(r) = \varepsilon_1 E(r)$$

$$D_2(r) = \varepsilon_2 E(r)$$

$$\oint_{S_z} \vec{D} \cdot \vec{dS} = Q \Rightarrow D_1(r) \cdot 2\pi r^2 + D_2(r) \cdot 2\pi r^2 = Q \Rightarrow E(r) = \frac{Q}{2\pi r^2 \varepsilon_0 (\varepsilon_{r1} + \varepsilon_{r2})}$$

$$\varepsilon_0 \varepsilon_{r1} \cdot E(r) \cdot 2\pi r^2 + \varepsilon_0 \varepsilon_{r2} E(r) \cdot 2\pi r^2 = Q$$

$$E(r) = \frac{Q}{2\pi \varepsilon_0 r^2 (\varepsilon_{r1} + \varepsilon_{r2})} \quad a < r < b$$

(6)

$$D_1(r) = \frac{\varepsilon_{r1}}{\varepsilon_{r1} + \varepsilon_{r2}} \frac{Q}{2\pi r^2} \quad a < r < b$$

$$D_2(r) = \frac{\varepsilon_{r2}}{\varepsilon_{r1} + \varepsilon_{r2}} \frac{Q}{2\pi r^2} \quad a < r < b$$

b) По услову задатка $E_{max} = E(r = a) = 350 \text{ kV/m}$,
одакле следи:

$$\frac{Q}{2\pi r^2 \varepsilon_0 (\varepsilon_{r1} + \varepsilon_{r2})} = E_{max} \Rightarrow Q = 2\pi \varepsilon_0 (\varepsilon_{r1} + \varepsilon_{r2}) a^2 E_{max}$$

$$Q = \frac{11}{2 \cdot 9 \cdot 10^9} \cdot 1 \cdot 10^{-4} \cdot 350 \cdot 10^3$$

$$Q \cong 21{,}4 \text{ nC}$$

(6)

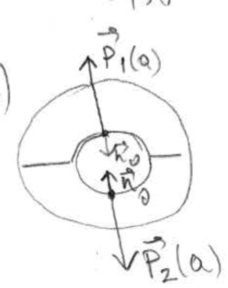
c) $$U_{PN} = \int_P^N \vec{E} \cdot \vec{dl} = \int_{r=a}^{r=b} E(r)\, dr = \frac{Q}{2\pi \varepsilon_0 (\varepsilon_{r1} + \varepsilon_{r2})} \int_a^b \frac{dr}{r^2} = \frac{Q}{2\pi \varepsilon_0 (\varepsilon_{r1} + \varepsilon_{r2})} \cdot \frac{b - a}{ab}$$

$$U_{PN} = \frac{\cancel{2\pi \varepsilon_0 (\varepsilon_{r1} + \varepsilon_{r2})} \cdot a^{\cancel{2}} \cdot E_{max}}{\cancel{2\pi \varepsilon_0 (\varepsilon_{r1} + \varepsilon_{r2})}} \cdot \frac{b - a}{\cancel{a} b} = \frac{a(b - a)}{b} \cdot E_{max} = \frac{1{,}5 \cdot 10^{-2}}{2{,}5} \cdot 350 \text{ kV}$$

(5)

$$U_{PN} = 2{,}1 \text{ kV}$$

d)

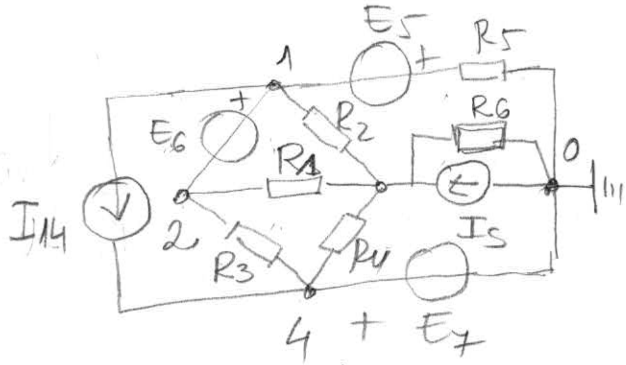


$$\sigma_{\sigma 1} = -P_1(a) = -\varepsilon_0 (\varepsilon_{r1} - 1) E_{max} = -\varepsilon_0 E_{max} \cong -3{,}098 \,\mu\text{C/m}^2$$

(6)

$$\sigma_{\sigma 2} = -P_2(a) = \underbrace{-\varepsilon_0 (\varepsilon_{r2} - 1)}_{-8\varepsilon_0} \cdot E_{max} = -24{,}78 \,\mu\text{C/m}^2$$

$$R = \frac{U_{14}}{I_{14}}$$



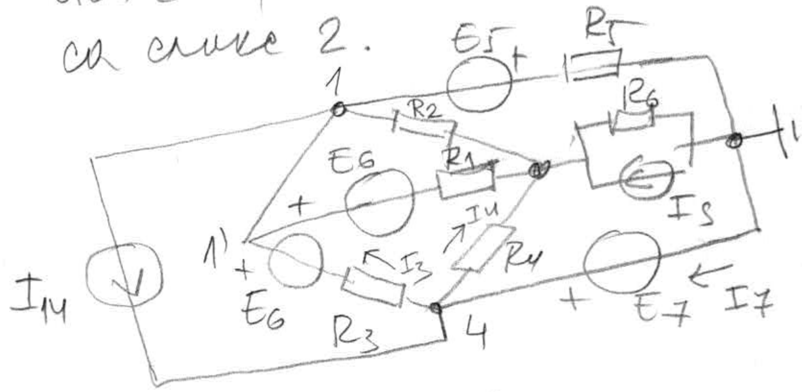
сл. 1

После примене теореме компенз. добија се мрежа са слике 1.

м.к.s $n = 6-2=4$

м.р.п $n = 4-2=2$

После премештања ідеал. емс E_6 добија се мрежа са слике 2.



слика 2

$E_5 = 12V$ $R_1 = R_2 = 100\Omega$

$E_6 = E_7 = 6V$ $R_3 = R_5 = 300\Omega$

$I_S = 120mA$ $R_4 = R_6 = 200\Omega$

$$V_4 = E_7 = 6V$$

$$\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_5}\right)V_1 - \left(\frac{1}{R_1}+\frac{1}{R_2}\right)V_3 - \frac{1}{R_3}V_4 = \frac{E_6}{R_1}+\frac{E_6}{R_3}-\frac{E_5}{R_5}-I_{14}$$

$$-\left(\frac{1}{R_1}+\frac{1}{R_2}\right)V_1 + \left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_4}+\frac{1}{R_6}\right)V_3 - \frac{1}{R_4}V_4 = -\frac{E_6}{R_1}+I_S$$

$$\left(\frac{2}{100}+\frac{2}{300}\right)V_1 - \frac{2}{100}V_3 = \frac{6}{300}+\frac{6}{100}+\frac{6}{300}-\frac{12}{300}-20m \quad /\cdot 300$$

$$-\frac{2}{100}V_1 + \left(\frac{2}{100}+\frac{2}{200}\right)V_3 = \frac{6}{200}-\frac{6}{100}+120m \quad /\cdot 100$$

$$\left.\begin{aligned} 8V_1 - 6V_3 &= 12 \\ -2V_1 + 3V_3 &= 9 \;/\cdot 2 \end{aligned}\right\} + \Rightarrow 4V_1 = 30 \quad \boxed{V_1 = 7{,}5V} \qquad 3V_3 = 9+2\cdot 7{,}5 \quad \boxed{V_3 = 8V}$$

a) $U_{14} = V_1 - V_4 = 1{,}5A$

$$R = \frac{U_{14}}{I_{14}} = \frac{1{,}5}{20m} = 75\Omega$$

$$\boxed{R = 75\Omega}$$

b) $P_S = U_S I_S = V_3 I_S = 0{,}96W$

$P_E = E_7 I_7$

$$I_7 = I_3 + I_4 - I_{14} = \frac{\overset{6}{V_4}-\overset{7,5}{V_1}+\overset{6}{E_6}}{\underset{300}{R_3}} + \frac{\overset{6}{V_4}-\overset{8}{V_3}}{\underset{200}{R_4}} - \overset{20m}{I_{14}}$$

$$I_7 = 15m - 10m - 20m = -15mA$$

$$\boxed{P_E = -90mW}$$

Ispit, 28.01.2015.

II-1

$E_5 = 12\,V$

$E_6 = E_7 = 6\,V$

$I_S = 120\,mA$

$R_1 = R_2 = 100\,\Omega$

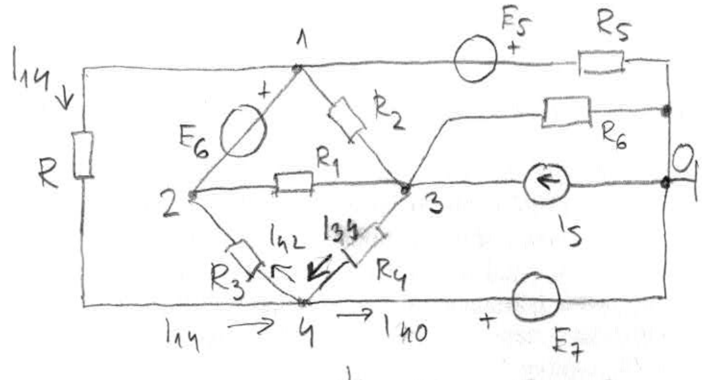
$R_3 = R_5 = 300\,\Omega$

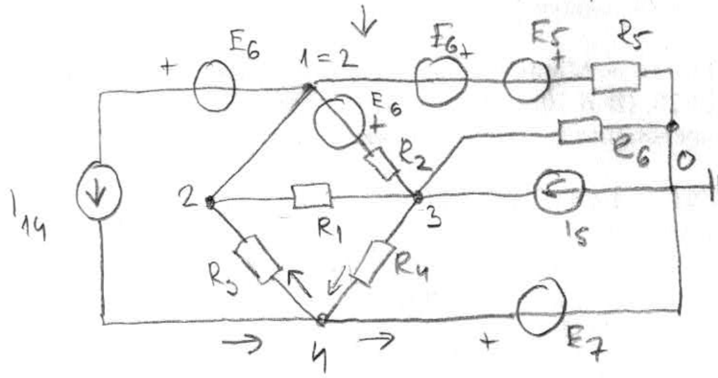
$R_4 = R_6 = 200\,\Omega$

$I_{14} = 20\,mA$

$R = ?$

$P_{I_S}, P_{E_7} = ?$





Primena teoreme o kompenzaciji + pomeranje idealnog naponskog generatora

$n_g = 9 \quad n_č = 4$

MKS: $n_g - (n_č - 1) - n_s = 9 - 3 - 2 = 4$

MPČ $= n_č - 1 - n_g = 4 - 1 - 1 = 2$

Č4: $V_4 = E_7 = 6V$

Č3: $V_3 \cdot \left(\frac{1}{R_2+0} + \frac{1}{R_1} + \frac{1}{R_4} + \underbrace{\cancel{\frac{1}{\infty}}}_{0} + \frac{1}{R_6} \right) - V_2 \cdot \left(\frac{1}{R_2+0} + \frac{1}{R_1} \right) - V_4 \cdot \frac{1}{R_4} = \frac{E_6}{R_2} + I_s$

Č2: $V_2 \cdot \left(\frac{1}{R_1} + \frac{1}{R_3} + \underbrace{\cancel{\frac{1}{0+\infty}}}_{0} + \frac{1}{0+R_2} + \frac{1}{0+0+R_5} \right) - V_3 \cdot \left(\frac{1}{R_1} + \frac{1}{0+R_2} \right) - V_4 \cdot \left(\underbrace{\cancel{\frac{1}{0+\infty}}}_{0} + \frac{1}{R_3} \right) = -\frac{E_6}{R_2} - \frac{E_6+E_5}{R_5} - I_{14}$

$V_3 \cdot \left(\underset{100}{\frac{1}{R_2}} + \underset{100}{\frac{1}{R_1}} + \underset{200}{\frac{1}{R_4}} + \underset{200}{\frac{1}{R_6}} \right) - V_2 \cdot \left(\underset{100}{\frac{1}{R_2}} + \underset{100}{\frac{1}{R_1}} \right) - V_4 \cdot \underset{200}{\frac{1}{R_4}} = \underset{100}{\frac{E_6}{R_2}} + I_s \;/\cdot 200$

$V_2 \cdot \left(\underset{100}{\frac{1}{R_1}} + \underset{300}{\frac{1}{R_3}} + \underset{100}{\frac{1}{R_2}} + \underset{300}{\frac{1}{R_5}} \right) - V_3 \cdot \left(\underset{100}{\frac{1}{R_1}} + \underset{100}{\frac{1}{R_2}} \right) - V_4 \cdot \underset{300}{\frac{1}{R_3}} = -\underset{100}{\frac{E_6}{R_2}} - \underset{300}{\frac{E_6+E_5}{R_5}} - I_{14} \;/\cdot 300$

$V_3 \cdot (2 + 2 + 1 + 1) - V_2 \cdot (2+2) - V_4 = 2E_6 + 200 I_s$

$V_2 \cdot (3 + 1 + 3 + 1) - V_3 \cdot (3+3) - \underset{6}{V_4} = -3\underset{6}{E_6} - \underset{6}{E_6} - \underset{12}{E_5} - 300 I_{14}$

$6V_3 - 4V_2 = 12 + 24 + 6$

$8V_2 - 6V_3 = -24 - 12 + 6 - 6$

$\left.\begin{aligned} -4V_2 + 6V_3 &= 42 \\ 8V_2 - 6V_3 &= -36 \end{aligned}\right\} +$

$4V_2 = 6 \Rightarrow \boxed{\begin{aligned} V_2 &= 1{,}5\,V, \\ V_3 &= 8V \end{aligned}} \quad V_3 = \frac{42 + 4V_2}{6} = \frac{42+6}{6} = 8\,V$

$U_{12} = V_1 - V_2 = E_6 \Rightarrow V_1 = V_2 + E_6 = 1{,}5 + 6 = 7{,}5\,V$

$U_{14} = V_1 - V_4 = 7{,}5 - 6 = 1{,}5\,V$

$$\boxed{R = \frac{U_{14}}{I_{14}} = \frac{1{,}5}{20 \cdot 10^{-3}} = 75\,\Omega}$$

$U_{IS} = U_{30} = V_3 = 8\,V$

$$\boxed{P_{IS} = U_{IS} I_S = 8 \cdot 120 \cdot 10^{-3} = 0{,}96\,W}$$

Č4: $-I_{14} + I_{42} - I_{34} + I_{40} = 0$

$$-I_{14} + \frac{V_4 - V_2}{R_3} - \frac{V_3 - V_4}{R_4} + I_{40} = 0$$

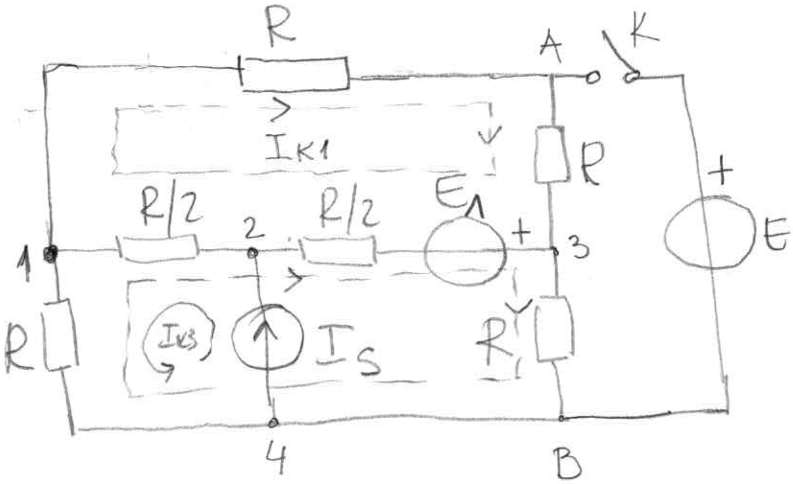
$$-20 \cdot 10^{-3} + \frac{6 - 1{,}5}{300} - \frac{8 - 6}{200} + I_{40} = 0$$

$$-20 \cdot 10^{-3} + 15 \cdot 10^{-3} - 10 \cdot 10^{-3} + I_{40} = 0$$

$$I_{40} = (20 - 15 + 10) \cdot 10^{-3} = 15\,mA$$

$P_{E7} = -E_7 \cdot I_{40} = -6 \cdot 15 \cdot 10^{-3} = -90\,mW$

II-2. Испит 28.01.2015.



$$E_T = U_{AB} = RI_{k1} + RI_{k2} = 5V$$

$E_T = ?$ мкс : $n = 2$

$E_T = U_{AB}$ мпч : $n = 3$

$$3RI_{k1} - RI_{k2} + 0{,}5RI_S = -E_1$$

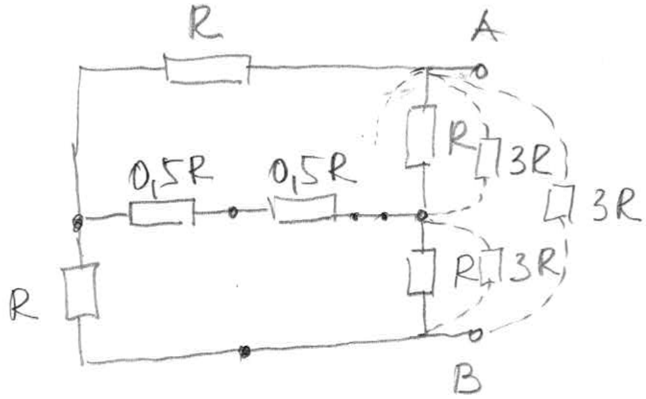
$$-RI_{k1} + 3RI_{k2} - 1{,}5RI_S = E_1$$

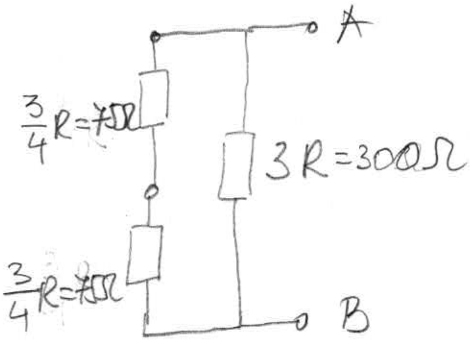
$$3I_{k1} - I_{k2} = -\frac{E_1}{R} - 0{,}5I_S = -170m$$

$$-I_{k1} - 3I_{k2} = \frac{E_1}{R} + 1{,}5I_S = 270m$$

$$8I_{k1} = -240m \Rightarrow \boxed{I_{k1} = -30mA}$$

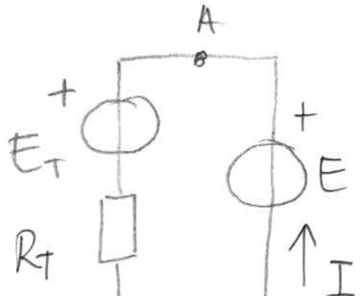
$$I_{k2} = 3I_{k1} + 170 \Rightarrow \boxed{I_{k2} = 80mA}$$





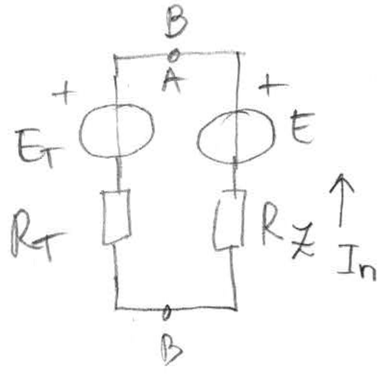
$$R_T = \frac{150 \cdot 300}{450}$$

$$\boxed{R_T = 100\Omega}$$



$$I = \left|\frac{E - E_T}{R_T}\right| = 50mA > I_{max}$$

Треба прикључити заштитни отп.



$$I_n = \left|\frac{E - E_T}{R_T + R_Z}\right| \leq I_{max} \Rightarrow R_Z = \frac{|E - E_T|}{I_{max}} - R_T$$

$$R_Z = \frac{5}{25m} - 100 = 100\Omega \quad P_{max_Z} \geq R_Z I_{max}^2$$

$$P_{max} \geq 62{,}5\,mW$$