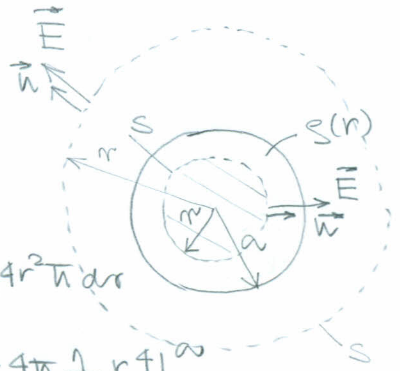
1a) $\rho(r) = \frac{\rho_0}{a} r$



$\oint_S \vec{E} \cdot d\vec{S} = \frac{1}{\varepsilon_0} \int \rho(r)\, dV$

$r < a$

$E \cdot 4r^2\pi = \frac{1}{\varepsilon_0} \int_0^{r} \frac{\rho_0}{a} r\, 4r^2\pi\, dr$

$E \cdot 4r^2\cancel{\pi} = \frac{1}{\varepsilon_0} \frac{\rho_0}{a} \cancel{4\pi} \frac{1}{\cancel{4}} r^4 \Big|_0^r$

$E \cdot 4\cancel{r^2} = \frac{\rho_0}{\varepsilon_0 a} r^{\cancel{4}\,2}$

$\boxed{E_1(r) = \frac{\rho_0}{4\varepsilon_0 a} r^2}$ $r < a$

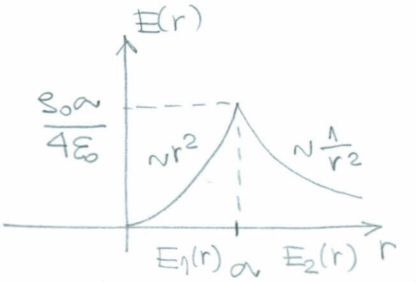
$r > a$

$E \cdot 4r^2\pi = \frac{1}{\varepsilon_0} \int_0^{a} \frac{\rho_0}{a} 4r^2\pi\, dr$

$E \cdot 4r^2\pi = \frac{1}{\varepsilon_0} \frac{\rho_0}{a} 4\pi \frac{1}{4} r^4 \Big|_0^a$

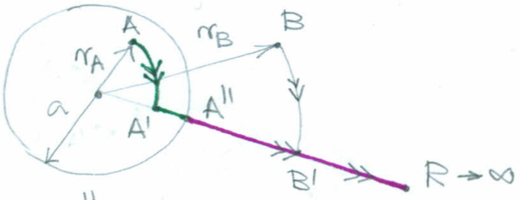
$E \cdot 4r^2\cancel{\pi} = \frac{\rho_0 \cancel{4\pi}}{\cancel{4}\varepsilon_0 \cancel{a}} a^{\cancel{4}\,3}$

$\boxed{E_2(r) = \frac{\rho_0 a^3}{4\varepsilon_0 r^2}}$ $r > a$



(10)

b)



$V_A = \int_A^{A'} \vec{E}_1 \cdot d\vec{l} + \int_{A'}^{A''} \vec{E}_1 \cdot d\vec{l} + \int_{A''}^{R} \vec{E}_2 \cdot d\vec{l} = \int_{r_A}^{a} \frac{\rho_0}{4\varepsilon_0 a} r^2\, dr + \int_a^{\infty} \frac{\rho_0 a^3}{4\varepsilon_0 r^2}\, dr =$

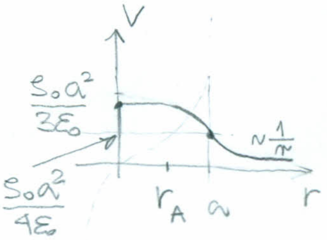
$\int_A^{A'} \vec{E}_1 \cdot d\vec{l} \to 0$, $d\vec{l} \perp \vec{E}$; $d\vec{l} = d\vec{r}$

$= \frac{\rho_0}{4\varepsilon_0 a} \frac{1}{3} r^3 \Big|_{r_A}^{a} + \frac{\rho_0 a^3}{4\varepsilon_0} \left(-\frac{1}{r}\right)\Big|_a^{\infty} = \frac{\rho_0}{12\varepsilon_0 a}(a^3 - r_A^3) + \frac{\rho_0 a^{\cancel{3}\,2}}{4\varepsilon_0 \cancel{a}}$

$\boxed{V_A = \frac{\rho_0 a^2}{3\varepsilon_0} - \frac{\rho_0 r_A^3}{12\varepsilon_0 a}}$ $r < a$

$V_B = \int_B^{B'} \vec{E}_2 \cdot d\vec{l} + \int_{B'}^{R \to \infty} \vec{E}_2 \cdot d\vec{l} = \int_{r_B}^{\infty} \frac{\rho_0 a^3}{4\varepsilon_0 r^2}\, dr = \frac{\rho_0 a^3}{4\varepsilon_0} \left(-\frac{1}{r}\right)\Big|_{r_B}^{\infty}$

$\int_B^{B'} \vec{E}_2 \cdot d\vec{l} \to 0$



$\boxed{V_B = \frac{\rho_0 a^3}{4\varepsilon_0 r_B}}$ $r > a$

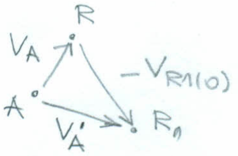
(8)

c) $A = Q_p V_B (r = 3a) = Q_p \cdot \frac{\rho_0 a^{\cancel{3}\,2}}{4\varepsilon_0 \cdot 3\cancel{a}}$

$\rho_0 = \frac{12\varepsilon_0 A}{Q_p a^2} = \frac{12\varepsilon_0 \cdot 10 \cdot \cancel{10^{-12}}}{1 \cdot \cancel{10^{-12}} \cdot 0{,}05^2} = 0{,}425\, \mu C/m^3$ $\left(424{,}8\ \frac{nC}{m^3}\right)$ ✓

(4)

d) $R(\infty) \to R_1(0)$

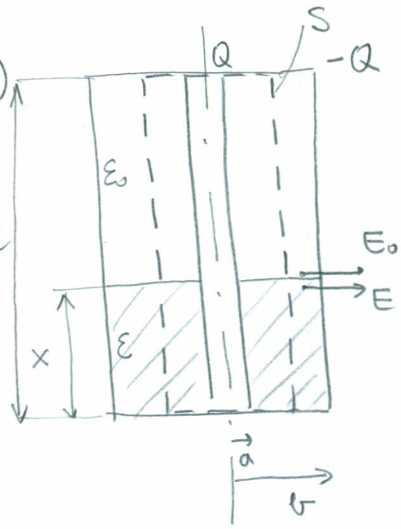


$V_A = \int_A^R \vec{E} \cdot d\vec{l}$

$V_A' = \int_A^{R_1(0)} \vec{E} \cdot d\vec{l} = \int_A^R \vec{E} \cdot d\vec{l} + \int_R^{R_1} \vec{E} \cdot d\vec{l} = V_A - \boxed{\int_{R_1(0)}^{R} \vec{E} \cdot d\vec{l}}$ $\longrightarrow$ smanji se za: $\boxed{\Delta V = \frac{\rho_0 a^2}{3\varepsilon_0}}$ $\sim 40V$

(3)

2



$$E_0 = E$$

$$\oint_S \vec{D}\cdot d\vec{S} = Q_{us}$$

$$\underbrace{D_0}_{\varepsilon_0 E}\cdot 2\pi r\cdot(l-x) + \underbrace{D}_{\varepsilon E}\cdot 2\pi r x = Q$$

$$E = \frac{Q}{2\pi\varepsilon_0[l+(\varepsilon_r-1)x]r}, \quad a<r<b$$

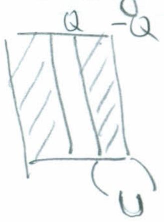
$$D_0 = \varepsilon_0 E \quad , \quad D = \varepsilon E$$

$$U_N = \int_a^b E(r)\cdot dr = \frac{Q}{2\pi\varepsilon_0[l+(\varepsilon_r-1)x]}\ln\frac{b}{a}$$

$$U_N = \frac{Q}{C(x)}$$

$$C(x) = \frac{Q}{U_N} = \frac{2\pi\varepsilon_0[l+(\varepsilon_r-1)x]}{\ln\frac{b}{a}}$$

Pre ispuštanja dielektrika:



$$C_{STARO} = C(x=l) = \frac{2\pi\varepsilon\cdot l}{\ln\frac{b}{a}} = \frac{2\pi\, 3\cdot 8{,}85\cdot 10^{-12}}{\ln 2} = 240{,}6\,\text{pF}$$

$$Q = C_{STARO}\cdot U$$

a) $$E_{max} = E(r=a) = \frac{Q}{2\pi\varepsilon_0[l+(\varepsilon_r-1)x]a} = \frac{C_{STARO}\cdot U}{2\pi\varepsilon_0[l+(\varepsilon_r-1)x]a}$$

b) za $x = 25\,\text{cm}$ $\quad E_{max} = E_{\check{c}o} \leftarrow \min\{E_{\check{c}o}, E_{\check{c}d}\}$

$$\frac{C_{STARO}\cdot U}{2\pi\varepsilon_0[l+(\varepsilon_r-1)x]a} = E_{\check{c}o}$$

$$\longrightarrow Q = E_{\check{c}o}\, 2\pi\varepsilon_0(l+2x)\cdot a = 3\cdot 10^6\cdot\frac{1}{2}\frac{1}{9\cdot 10^9}\cdot 1{,}5\cdot 2\cdot 10^{-3}$$
$$= 0{,}5\cdot 10^{-6}\,\text{C}$$
$$= 500\,\text{nC}$$

$$\frac{\frac{\cancel{2\pi\varepsilon_0}\cdot\varepsilon_r\cdot l}{\ln b/a}\cdot U}{\cancel{2\pi\varepsilon_0}[l+(\varepsilon_r-1)x]a} = E_{\check{c}o} \implies U = E_{\check{c}o}\frac{[l+(\varepsilon_r-1)x]\cdot a}{\varepsilon_r\cdot l}\ln\frac{b}{a}$$

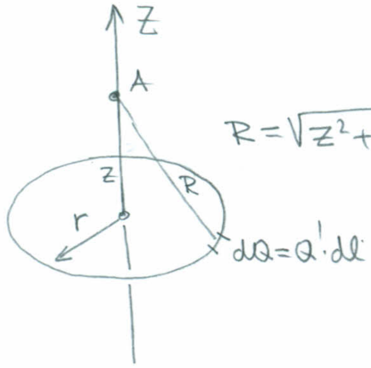
$$= 3\cdot 10^6\,\frac{\cancel{[1+2\cdot 0{,}25]}\,2\cdot 10^{-3}}{\cancel{3}}\ln 2 = 3\ln 2\cdot\text{kV}$$
$$= 2{,}08\,\text{kV}$$

c)
$$\rho_v(r=a) = -P(r=a) = -(\varepsilon-\varepsilon_0)E(r=a) = -\varepsilon_0(\varepsilon_r-1)E_{\check{c}o} = -2\varepsilon_0\cdot E_{\check{c}o}$$

$$Q_v(r=a) = \rho_v(r=a)\cdot 2\pi a\cdot x = -2\varepsilon_0 E_{\check{c}o}\cdot 2\pi a\cdot x =$$
$$= -2\cdot 8{,}85\cdot 10^{-12}\cdot 3\cdot 10^6\cdot 2\pi\cdot 2\cdot 10^{-3}\cdot 0{,}25 = -166{,}7\,\text{nC}$$
$$-\frac{Q}{3}$$

a) 10
b) 10
c) 5

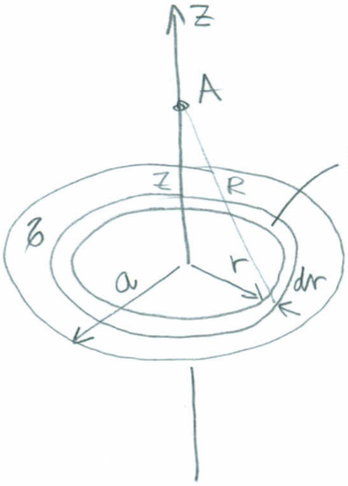
1. a)



$R=\sqrt{z^2+r^2}$

$$dV=\frac{dQ}{4\pi\varepsilon_0 R}$$ ← za tačkasto naelektrisanje ako je ref. tačka u ∞

$$V=\int\limits_{\text{po prstenu}} dV = \frac{Q'}{4\pi\varepsilon_0 R}\int_0^{2\pi r} dl = \frac{Q'\cdot 2\pi r}{4\pi\varepsilon_0 R}=\frac{Q_{PRSTENA}}{4\pi\varepsilon_0 R}$$



$dQ=\sigma\cdot dS$

$dS=2\pi r\cdot dr$

$\sigma=\text{const.}$

za prsten:

$$dV=\frac{dQ}{4\pi\varepsilon_0 R}=\frac{\sigma\cdot\cancel{2\pi} r\,dr}{_2\cancel{4\pi}\varepsilon_0\sqrt{z^2+r^2}}$$

$$V=\int\limits_{\text{po disku}} dV=\frac{\sigma}{2\varepsilon_0}\int_0^a\frac{r\,dr}{\sqrt{z^2+r^2}}$$

smena $t^2=z^2+r^2$

$\cancel{2}t\,dt=\cancel{2}r\,dr$

$$V=\frac{\sigma}{2\varepsilon_0}\int_{|z|}^{\sqrt{z^2+a^2}}\frac{\cancel{t}\,dt}{\cancel{t}}$$

$$V=\frac{\sigma}{2\varepsilon_0}\left(\sqrt{z^2+a^2}-|z|\right)=V(z)$$

b)

$$E_z=-\frac{\partial V}{\partial z}=-\frac{\sigma}{2\varepsilon_0}\left(\frac{1}{\cancel{2}}\cancel{2}z\cdot(z^2+a^2)^{-\frac{1}{2}}-1\right)\qquad \vec{E}(z)=E_z\cdot\vec{i}_z$$

$$E_A=E_z(a)=-\frac{\sigma}{2\varepsilon_0}\left(\frac{\cancel{a}}{\cancel{a}\sqrt{2}}-1\right)=\frac{\sigma}{2\varepsilon_0}\left(1-\frac{\sqrt{2}}{2}\right)$$

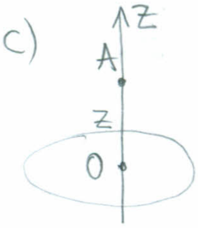
$$|F|=Q_p\cdot|E_A|=\frac{Q_p\cdot|\sigma|}{2\varepsilon_0}\,\frac{2-\sqrt{2}}{2}\longrightarrow|\sigma|=\frac{4\varepsilon_0|F|}{Q_p\,(2-\sqrt{2})}$$

$$|\sigma|=\frac{4\cdot 8{,}85\cdot\cancel{10^{-12}}\cdot 10\cdot 10^{-6}}{\cancel{10^{-12}}\cdot(2-\sqrt{2})}=610{,}3\,\mu C/m^2$$

Sila $\vec{F}$ je privlačna i $Q_p>0$ onda $\sigma<0$.

$$Q_{uk}=-|\sigma|\cdot a^2\pi=-610{,}3\cdot10^{-6}\cdot(5\cdot10^{-2})^2\pi=-4{,}79\,\mu C$$

c)



$\cdot R\to\infty$

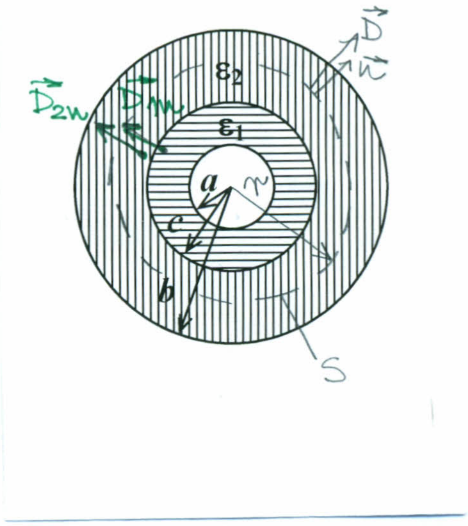
a) 10
b) 10
c) 5

$$V_{Novo}(z)=V(z)+U_{R0}=V(z)+\cancel{V_R}^{\infty}-V_0$$

$$V_{Novo}(z)-V(z)=-V_0=-V(z=0)=-\frac{\sigma}{2\varepsilon_0}a=-\frac{-610{,}3\cdot10^{-6}\cdot5\cdot10^{-2}}{2\cdot8{,}85\cdot10^{-12}}=172{,}4\cdot10^4\,V$$

B2

a) $\oint_S \vec{D}\cdot d\vec{s} = Q$ GR. USLOV $D_{1n} = D_{2n}$



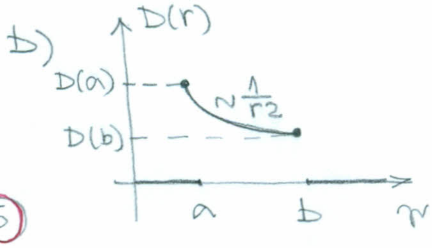
$D \cdot 4r^2\pi = Q$

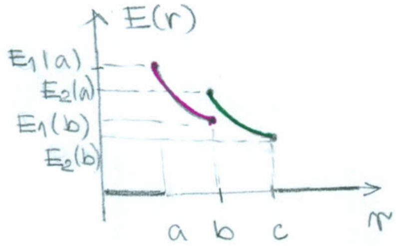
$$\boxed{D = \frac{Q}{4r^2\pi}} \quad a<r<b$$

$$E_1 = \frac{D}{\varepsilon_1} = \frac{Q}{4r^2\pi\varepsilon_1} \quad a<r<c$$

3 $$E_2 = \frac{D}{\varepsilon_2} = \frac{Q}{4r^2\pi\varepsilon_2} \quad c<r<b$$

b)





5

c) $$U = \int_{+}^{-} \vec{E}\cdot d\vec{l} = \int_a^c \underbrace{\frac{Q}{4r^2\pi\varepsilon_1}}_{E_1}\underbrace{dr}_{dl} + \int_c^b \underbrace{\frac{Q}{4r^2\pi\varepsilon_2}}_{E_2}\underbrace{dr}_{dl} = \frac{Q}{4\pi\varepsilon_1}\left(-\frac{1}{r}\right)\Big|_a^c + \frac{Q}{4\pi\varepsilon_2}\left(-\frac{1}{r}\right)\Big|_c^b$$

4 $$U = \frac{Q}{4\pi\varepsilon_1}\left(\frac{1}{a}-\frac{1}{c}\right) + \frac{Q}{4\pi\varepsilon_2}\left(\frac{1}{c}-\frac{1}{b}\right) = \boxed{\frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{\varepsilon_{r1}}\frac{c-a}{ac} + \frac{1}{\varepsilon_{r2}}\frac{b-c}{cb}\right)}$$

4 $$E_{1m}(r=a) = \frac{Q_{1m}}{4a^2\pi\varepsilon_1} \le E_{č1} \rightarrow \boxed{Q_{1m} = 4a^2\pi\varepsilon_1 E_{č1} = 4\cdot 0{,}03^2\pi\cdot 4\varepsilon_0\cdot 9\cdot 10^6 = 3{,}6\,\mu C}$$

$$E_{2m}(r=c) = \frac{Q_{2m}}{4c^2\pi\varepsilon_2} \le E_{č2} \rightarrow \boxed{Q_{2m} = 4c^2\pi\varepsilon_2 E_{č2} = 4\cdot 0{,}06^2\pi\, 2{,}5\varepsilon_0\cdot 12\cdot 10^6 = 12\,\mu C}$$

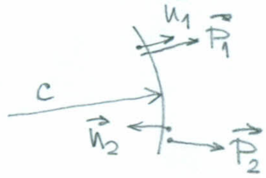
$Q_m = \min(Q_{1m}, Q_{2m}) = Q_{1m} \rightarrow$ UNUTRAŠNJI SLOJ JE KRITIČNIJI
DO PROBOJA ĆE DOĆI NA POLUPR. $r = a$

3 $$\boxed{U_p = \underbrace{\frac{Q_m}{4\pi\varepsilon_0}}_{32400}\left(\frac{1}{\underset{4}{\varepsilon_{r1}}}\frac{\overset{3cm}{c-a}}{\underset{6cm}{ac}} + \frac{1}{\underset{2{,}5}{\varepsilon_{r2}}}\frac{\overset{6cm}{b-c}}{\underset{12cm}{cb}}\right)} = \boxed{242{,}8\text{ kV}}$$

4,167 3,33

7.5

d) $Q_V = Q_{V1} + Q_{V2} = \sigma_{V1}\cdot S + \sigma_{V2}\cdot S = (\sigma_{V1}+\sigma_{V2})\cdot S$



$$\sigma_{V1} = \vec{P}_1\cdot\vec{n}_1 = P_1(c) = D(c)\left(1-\frac{1}{\underset{4}{\varepsilon_{r1}}}\right) = \frac{3}{4}D(c) = \frac{3}{4}\frac{Q}{4c^2\pi}$$

$$\sigma_{V2} = \vec{P}_2\cdot\vec{n}_2 = -P_2(c) = -D(c)\left(1-\frac{1}{\underset{2{,}5}{\varepsilon_{r2}}}\right) = -\frac{3}{5}D(c) = -\frac{3}{5}\frac{Q}{4c^2\pi}$$

$$P = D - \varepsilon_0 E = D\left(1-\frac{1}{\varepsilon_r}\right)$$

6 $$\boxed{Q_V = \left(\frac{3}{4}\frac{Q}{\cancel{4c^2\pi}} - \frac{3}{5}\frac{Q}{\cancel{4c^2\pi}}\right)\cdot\cancel{4c^2\pi} = \frac{3}{4}Q - \frac{3}{5}Q = 0{,}15Q}$$

$$U = \frac{U_p}{2} \Rightarrow Q = \frac{Q_m}{2}$$

$$\boxed{Q_V = 0{,}15\frac{Q_m}{2} = 0{,}15\frac{3{,}6\mu}{2} = \boxed{0{,}27\,\mu C}}$$

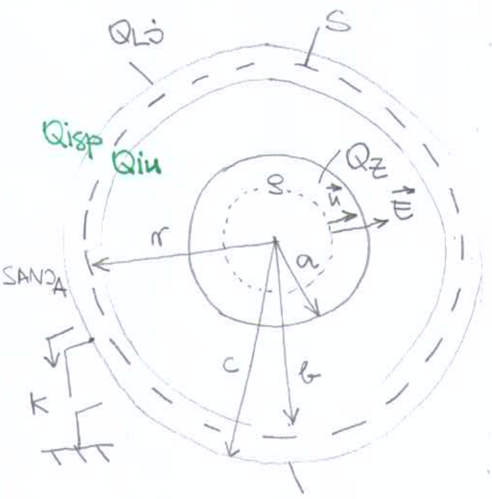
a)

- $\oint_S \vec{E}\cdot d\vec{s} = \frac{Q_{us}}{\varepsilon_0}$

$0 = \frac{1}{\varepsilon_0}(Q_z + Q_{in}) \rightarrow Q_{in} = -Q_z$

- NA OSNOVU ZAKONA ODRŽANJA KOLIČINE NAELEKTRISANJA

$Q_{Lj} = Q_{in} + Q_{isp} \rightarrow Q_{isp} = Q_{Lj} + Q_z$



NAELEKTRISANA PROVODNA LJUSKA

U ZEMLJU ODLAZI (PRI ZATVORENOM PREK.) NAELEKTRISANJE KOJE SE NALAZI NA SPOLJAŠNJOJ STRANI LJUSKE

$Q = Q_{isp} = Q_{Lj} + Q_z$

5 $Q_{Lj} = Q - Q_z = 0{,}48\,nC - \underbrace{\rho \frac{4}{3} a^3 \pi}_{Q_z} = 0{,}48\,nC - \underbrace{7\cdot 10^{-6}\frac{4}{3}0{,}03^3\pi}_{Q_z = 0{,}792\,nC}$

$\boxed{Q_{Lj} = -312\,pC}$ $(-0{,}312\,nC)$

b) $\sigma_u = \frac{Q_{in}}{4b^2\pi} = \frac{-0{,}792\,n}{4\cdot 0{,}05^2\pi} = -25{,}2\,nC/m^2$

5 $\sigma_{sp} = \frac{Q_{isp}}{4c^2\pi} = \frac{0{,}48\,n}{4\cdot 0{,}055^2\pi} = 12{,}63\,nC/m^2$

c) $\oint_S \vec{E}\cdot d\vec{s} = \frac{Q_{us}}{\varepsilon_0}$

$r<a$ $E\cdot 4r^2\pi = \frac{1}{\varepsilon_0}\int_0^{r!} \rho\, 4r^2\pi\, dr$

5 $\cancel{E\cdot 4r^2\pi} = \frac{1}{\varepsilon_0}\rho 4\pi \frac{1}{3} r^3$

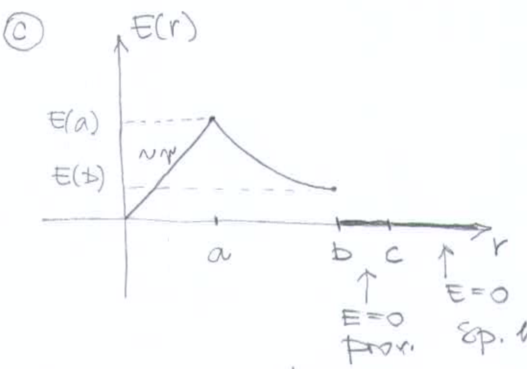
$\boxed{E(r) = \frac{\rho}{3\varepsilon_0} r}$

$a<r<b$ $E\cdot 4r^2\pi = \frac{1}{\varepsilon_0}\int_0^{a!} \rho\, 4r^2\pi\, dr$

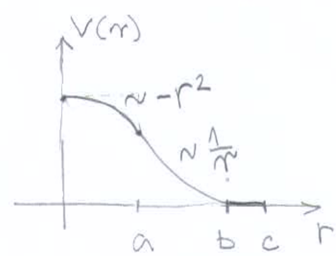
$E\cdot 4r^2\pi = \frac{1}{\varepsilon_0}\rho 4\pi \frac{1}{3} a^3$

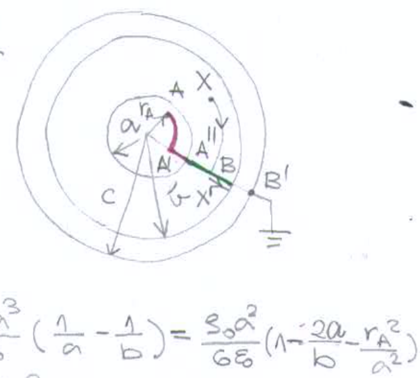
$\boxed{E(r) = \frac{\rho a^3}{3\varepsilon_0 r^2}}$

5 c)



prov. sp. ljuske je ⏚





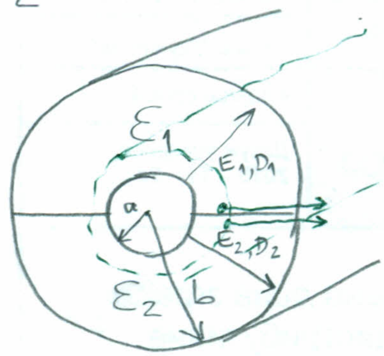
d) $r<a$ $V_A = \int_A^{A'} \vec{E}\cdot d\vec{l} + \int_{A'}^{A''} \vec{E}\cdot d\vec{l} + \int_{A''}^{B} \vec{E}\cdot d\vec{l} + \int_{B}^{B'} \vec{E}\cdot d\vec{l}$

5 $V_A = \int_{r_A}^{a} \frac{\rho}{3\varepsilon_0} r\, dr + \int_a^b \frac{\rho a^3}{3\varepsilon_0 r^2} dr = \frac{\rho}{2\varepsilon_0 6}\cdot\frac{a^2 - r_A^2}{2} + \frac{\rho a^3}{3\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{\rho_0 a^2}{6\varepsilon_0}\left(1 - \frac{2a}{b} - \frac{r_A^2}{a^2}\right)$

$a<r<b$ $V_X = \int_X^{X'} \vec{E}\cdot d\vec{l} + \int_{X'}^{B} E\cdot d\vec{l} + \int_B^{B'} \vec{E}\cdot d\vec{l} = \int_{r_{X'}}^{b} \frac{\rho a^3}{3\varepsilon r^2} dr = \frac{\rho a^3}{3\varepsilon_0}\left(\frac{1}{r_{X'}} - \frac{1}{b}\right)$

$b<r<c$ $V=0$

-2



$a = 5mm$ $\varepsilon_{r1} = 3$
$b = 2{,}7a$ $\varepsilon_{r2} = ?$
$L = 1m$ $E_{\bar{c}1} = 90kV/cm$
Q $E_{\bar{c}2} = 120kV/cm$

a) gr. uslovi $E_{1t} = E_{2t} \Rightarrow E_1 = E_2 = E$

$D_1 = \varepsilon_1 E, \quad D_2 = \varepsilon_2 E$

$$\oint_S \vec{D}\,d\vec{S} = Q_{sl.} \rightarrow D_1 \cdot r\pi L + D_2 r\pi L = Q' \cdot L$$

$$E = \frac{Q'}{r\pi(\varepsilon_1+\varepsilon_2)}, \quad a<r<b$$

$$U = \int_a^b E\,dr = \frac{Q'}{\pi(\varepsilon_1+\varepsilon_2)} \ln\frac{b}{a}, \quad \boxed{C' = \frac{\pi(\varepsilon_1+\varepsilon_2)}{\ln b/a}}$$

10

b) $Q_D = \sigma_D \cdot a\pi L, \quad \sigma_D = D_2(r=a) = \varepsilon_2 \cdot \dfrac{Q'}{a\pi(\varepsilon_1+\varepsilon_2)}$

$$Q_D = \frac{\varepsilon_2 Q}{\varepsilon_1+\varepsilon_2} = \frac{3}{4}Q \rightarrow \frac{\varepsilon_1+\varepsilon_2}{\varepsilon_2} = 4/3 \rightarrow \boxed{\varepsilon_{r2} = 3\varepsilon_{r1} = 9}$$

10

c) $$E_{max} = \frac{Q'_{max}}{a\pi(\varepsilon_1+\varepsilon_2)} \le \min\{E_{\bar{c}1}, E_{\bar{c}2}\} = 90\,kV/cm$$

$$Q'_{max} = C' \cdot U_{max} = \frac{\pi(\varepsilon_1+\varepsilon_2)}{\ln\frac{b}{a}} \cdot U_{max} \rightarrow U_{max} = a \cdot \ln\frac{b}{a} \cdot E_{\bar{c}1}$$

$$\boxed{U_{max} = 45\,kV}$$

5

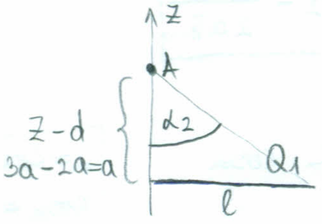
GRUPA D1

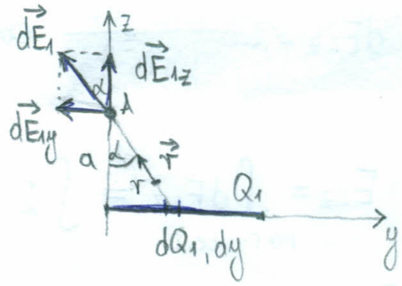
$\boxed{\text{I-1.}}$

$\boxed{Q_1' = \frac{Q_1}{\ell}}$

$Q_1 = 100\,nC$

a)

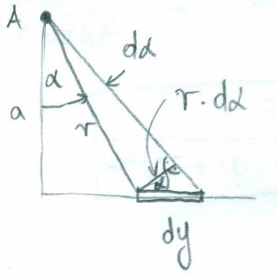




$$dE_1 = \frac{dQ_1}{4\pi\varepsilon_0 r^2} = \frac{Q_1' \cdot dy}{4\pi\varepsilon_0 r^2}$$

$$dE_{1z} = dE_1 \cdot \cos\alpha$$

$$dE_{1y} = dE_1 \cdot \sin\alpha$$



$$\cos\alpha = \frac{r \cdot d\alpha}{dy} \quad , \quad \underbrace{\cos\alpha \cdot dy = r \cdot d\alpha}_{} \; (*)$$

$$(**) \quad \boxed{\cos\alpha = \frac{a}{r}} \qquad 2$$

$$dE_{1z} = \frac{Q_1' \cdot dy^{(*)}}{4\pi\varepsilon_0 \cdot r^2} \cdot \cos\alpha = \frac{Q_1' \cdot \cancel{r} \cdot d\alpha}{4\pi\varepsilon_0 \cdot r^{\cancel{2}}_{(**)}} = \frac{Q_1' \cdot d\alpha}{4\pi\varepsilon_0 \cdot \frac{a}{\cos\alpha}} = \frac{Q_1'}{4\pi\varepsilon_0 \cdot a} \cdot \cos\alpha \cdot d\alpha$$

$$\boxed{E_{1z}} = \frac{Q_1'}{4\pi\varepsilon_0 \cdot a} \cdot \int_{\alpha_1 = 0}^{\alpha_2} \cos\alpha \cdot d\alpha = \frac{Q_1'}{4\pi\varepsilon_0 \cdot a} \left(\sin\alpha_2 - \cancelto{0}{\sin\alpha_1}\right) = \boxed{\frac{Q_1'}{4\pi\varepsilon_0 \cdot a} \cdot \frac{\ell}{\sqrt{a^2 + \ell^2}}} \qquad 5$$

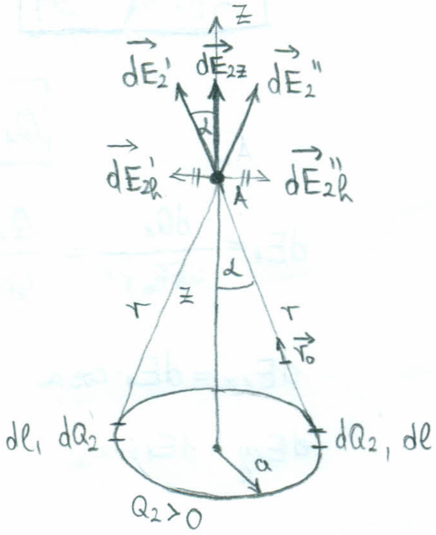
$$\boxed{\vec{E}_{1z} = E_{1z} \cdot \vec{i_z}}$$

$$dE_{1y} = \frac{Q_1' \cdot dy^{(*)}}{4\pi\varepsilon_0 \cdot r^2} \cdot \sin\alpha = \frac{Q_1' \cdot \frac{\cancel{r} \cdot d\alpha}{\cos\alpha}}{4\pi\varepsilon_0 \cdot r^{\cancel{2}}_{(**)}} \cdot \sin\alpha = \frac{Q_1' \cdot \frac{d\alpha}{\cancel{\cos\alpha}}}{4\pi\varepsilon_0 \cdot \frac{a}{\cancel{\cos\alpha}}} \cdot \sin\alpha = \frac{Q_1'}{4\pi\varepsilon_0 \cdot a} \cdot \sin\alpha \cdot d\alpha$$

$$\boxed{E_{1y}} = \frac{Q_1'}{4\pi\varepsilon_0 \cdot a} \cdot \int_{\alpha_1}^{\alpha_2} \sin\alpha \cdot d\alpha = \frac{Q_1'}{4\pi\varepsilon_0 \cdot a} \cdot \left(\cancelto{1}{\cos\alpha_1} - \cos\alpha_2\right) = \boxed{\frac{Q_1'}{4\pi\varepsilon_0 \cdot a} \cdot \left(1 - \frac{a}{\sqrt{a^2 + \ell^2}}\right)} \qquad 5$$

$$\boxed{\vec{E}_{1y} = E_{1y} \cdot (-\vec{i_y})}$$

b)



$$d\vec{E}_{2h}' + d\vec{E}_{2h}'' = 0$$

$$dE_{2z} = 2\cdot dE_2'\cdot\cos\alpha, \qquad dE_2' = \frac{dQ_2}{4\pi\varepsilon_0\cdot r^2} = \frac{Q_2'\cdot dl}{4\pi\varepsilon_0\cdot r^2}$$

$$dE_{2z} = 2\cdot\frac{Q_2'\cdot dl}{4\pi\varepsilon_0\cdot r^2}\cdot\cos\alpha \qquad \boxed{Q_2' = \frac{Q_2}{2a\pi}}$$

$$E_{2z} = \int_{\text{po prstenu}} dE_{2z} = \int 2\cdot\frac{Q_2'\cdot dl}{4\pi\varepsilon_0\cdot r^2}\cdot\cos\alpha \qquad \boxed{\begin{matrix}\alpha = \text{const.}\\ \cos\alpha = \frac{z}{r}\end{matrix}}$$

$$E_{2z} = 2\cdot\int_0^{\frac{1}{2}2a\pi}\frac{Q_2'\cdot\cos\alpha}{4\pi\varepsilon_0\cdot r^2}\,dl = \cancel{2}\cdot\frac{\frac{Q_2}{\cancel{2a\pi}}\cdot\cos\alpha}{4\pi\varepsilon_0\cdot r^2}\,\cancel{a\pi} = \frac{Q_2}{4\pi\varepsilon_0\cdot r^2}\cdot\frac{z}{r}$$

$$\boxed{z=3a} \qquad \boxed{r=\sqrt{a^2+z^2}}$$

$$\boxed{E_{2z}} = \frac{Q_2\cdot z}{4\pi\varepsilon_0\cdot r^3} = \boxed{\frac{Q_2\cdot z}{4\pi\varepsilon_0\cdot(a^2+z^2)^{3/2}}} \qquad \boxed{\vec{E}_{2z} = E_{2z}\cdot\vec{i}_z}$$

6

c)

$$\vec{E}_A = \vec{E}_1 + \vec{E}_2 = (E_{1z} + E_{2z})\cdot\vec{i}_z + E_{1y}\cdot(-\vec{i}_y)$$

$$E_{1z} + E_{2z} = 0 \qquad (Q_1>0 \Rightarrow \underline{\underline{Q_2<0}})$$

$$\frac{Q_1'}{\cancel{4\pi\varepsilon_0}\cdot a}\cdot\frac{l}{\sqrt{a^2+l^2}} = \frac{|Q_2|\cdot z}{\cancel{4\pi\varepsilon_0}\cdot(a^2+z^2)^{3/2}} \qquad \boxed{\begin{matrix}Q_1'\cdot l = Q_1\\ z=3a\end{matrix}}$$

4

$$\frac{Q_1}{a}\cdot\frac{1}{\sqrt{a^2+l^2}} = \frac{|Q_2|\cdot 3a}{(10a^2)^{3/2}}$$

$$|Q_2| = \frac{Q_1}{\cancel{a}}\cdot\frac{1}{\sqrt{a^2+l^2}}\cdot\frac{10\sqrt{10}\cdot a^{\cancel{3}\,1}}{3\cancel{a}} = \frac{Q_1\cdot a\cdot 10\sqrt{10}}{\sqrt{a^2+l^2}\cdot 3} \qquad \boxed{\begin{matrix}a=3\text{ cm}\\ l=5\text{ cm}\end{matrix}}$$

$$|Q_2| = 542{,}33\text{ nC}, \ (Q_2<0) \rightarrow \boxed{Q_2 = -542{,}33\text{ nC}}$$

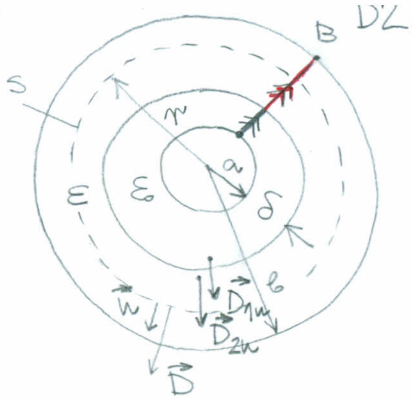
d)

$$\vec{E}_A = E_{1y}\cdot(-\vec{i}_y) = \frac{Q_1'}{4\pi\varepsilon_0\cdot a}\left(1-\frac{a}{\sqrt{a^2+l^2}}\right)\cdot(-\vec{i}_y) \qquad \boxed{\begin{matrix}a=3\text{ cm}\\ l=5\text{ cm}\end{matrix}}$$

$$\boxed{|\vec{E}_A|} = E_{1y} = \frac{\frac{Q_1}{l}}{4\pi\varepsilon_0\cdot a}\left(1-\frac{a}{\sqrt{a^2+l^2}}\right) = \boxed{291{,}18\ \frac{\text{kV}}{\text{m}}}$$

3

DZ



a) $\oint_S \vec{D}\cdot d\vec{S} = Q$ $\qquad D_{1n}=D_{2n}$

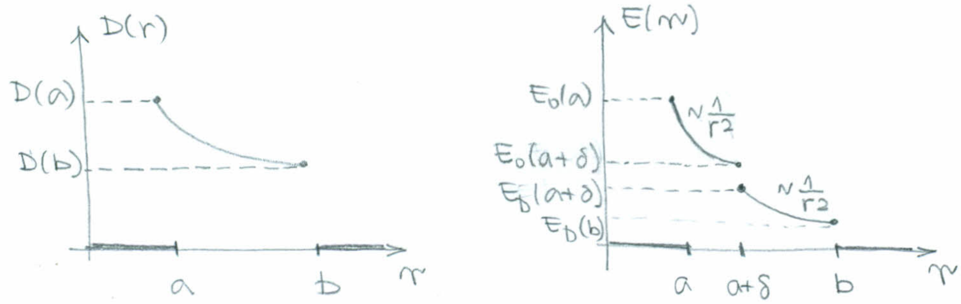
$D\cdot 4r^2\pi = Q$

$D=\frac{Q}{4r^2\pi}$ $\qquad a<r<b$

$E_0 = \frac{D}{\varepsilon_0} = \frac{Q}{4r^2\pi\varepsilon_0}$ $\qquad a<r<a+\delta$

(3) $E_D = \frac{D}{\varepsilon} = \frac{Q}{4r^2\pi\varepsilon_D}$ $\qquad a+\delta<r<b$

b)



(4)

c) $C_0=\frac{Q}{U}$

$U=\int_+^- \vec{E}\cdot d\vec{l} = \int_a^{a+\delta}\vec{E}_0\cdot d\vec{l} + \int_{a+\delta}^{b}\vec{E}_D\cdot d\vec{l} = \int_a^{a+\delta}\frac{Q}{4r^2\pi\varepsilon_0}\,dr + \int_{a+\delta}^{b}\frac{Q}{4r^2\pi\varepsilon}\,dr$

$U=\frac{Q}{4\pi\varepsilon_0}\left(-\frac{1}{r}\right)\Big|_a^{a+\delta} + \frac{Q}{4\pi\varepsilon}\left(-\frac{1}{r}\right)\Big|_{a+\delta}^{b} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{a+\delta}\right) + \frac{Q}{4\pi\varepsilon}\left(\frac{1}{a+\delta}-\frac{1}{b}\right)$

(5) $\left[C_0=\frac{Q}{U}=\frac{4\pi\varepsilon_0}{\left(\frac{1}{a}-\frac{1}{a+\delta}\right)+\frac{1}{\varepsilon_r}\left(\frac{1}{a+\delta}-\frac{1}{b}\right)}\right]$

$C_0=\frac{4\pi\varepsilon_0}{\left(\frac{1}{0,05}-\frac{1}{0,053}\right)+\frac{1}{4}\left(\frac{1}{0,053}-\frac{1}{0,1}\right)}$

$\boxed{C_0=33,2\,pF}$

d) $U_{m0}=\frac{Q_m}{C}=\frac{Q_{m0}}{C}$

$E_{0m}=\frac{Q_{0m}}{4a^2\pi\varepsilon_0}\le E_{\check{c}0} \to Q_{0m}=4a^2\pi\varepsilon_0 E_{\check{c}0}$

$E_{Dm}=\frac{Q_{Dm}}{4(a+\delta)^2\pi\varepsilon}\le E_{\check{c}D} \to Q_{Dm}=4(a+\delta)^2\pi\varepsilon E_{\check{c}D}$

$\Big\}\; Q_m=\min(Q_{0m},Q_{Dm})=\boxed{Q_{0m}}=0,834\,\mu C$

(5) $U_{m0}=\frac{0,834}{33,2\cdot10^{-12}}=\boxed{25,12\,kV}$

za pun kond. (bez vazdušnog zazora $\delta=0$)

$C=\frac{4\pi\varepsilon}{\frac{b-a}{ab}}=\boxed{\frac{4\pi\varepsilon ab}{b-a}}=\frac{4\pi4\varepsilon_0\cdot0,05\cdot0,1}{0,1-0,05}=\boxed{44,5\,pF}$

! $C_0\neq C$

sa vazd. slojem $\qquad$ bez vazd. sloja $\delta=0$!

$E_m=\frac{Q_m}{4a^2\pi\varepsilon}\le E_{\check{c}D}\to\boxed{Q_m}=4a^2\pi\varepsilon E_{\check{c}D}=16,68\,\mu C$

(5) $U_m=\frac{Q_m}{C}=\frac{16,68\,\mu C}{44,5\,pF}=\boxed{374,8\,kV}$

(3) $k=\frac{U_m}{U_{m0}}=14,9\sim15$ $\qquad$ U_m se smanji ~ 15 puta